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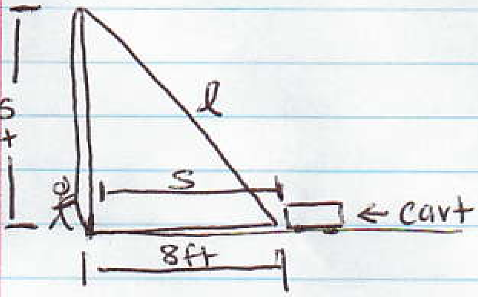
Ch. 3 (3.1|3.2) Sept. 30, 2010

Related Rates of Change

A rope is attached to a pulley mounted on a 15-ft tower. The end of the rope is attached to a heavily loaded cart. A worker can pull in rope at a rate of 2 ft/sec. How fast is the cart approaching the tower when it is 8 ft from the tower?

ex:

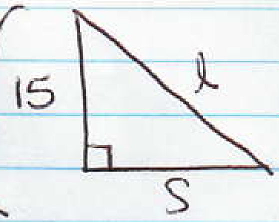
s is changing with time.



$$\frac{dl}{dt} = \frac{-2\text{ ft}}{\text{sec}}$$

$$\left|\frac{ds}{dt}\right| = ? \quad \text{when } s = 8\text{ft}.$$

will always be a right triangle



In pictures, label constants with #s; ~~table~~ label things that change (over time) with variables.

Find ~~formula(s)~~ equation(s) that are always true (usually involve geometry).

Differentiate these ~~formulas~~ equations (with respect to time).

$$15^2 + s^2 = l^2$$
$$d(15^2 + s^2) = d(l^2)$$
$$d(15^2) + d(s^2) = d(l^2)$$
$$\frac{0 + 2s\,ds}{dt} = \frac{2l\,dl}{dt}$$

Find l:

$$15^2 + 8^{ft} = l^2$$
$$225 + 64 = l^2$$
$$\sqrt{289} = \sqrt{l^2}$$
$$17 = l$$

When s is equal to 8ft:

$$2s\frac{ds}{dt} = 2l\frac{dl}{dt}$$
$$2(8)\frac{ds}{dt} = 2(17)\left(\frac{-2\text{ft}}{\text{sec}}\right)$$
$$\frac{16\frac{ds}{dt}}{16} = \frac{-68}{16}$$

Rate of change? $\frac{ds}{dt} = \frac{-17}{4}$

How fast is it approaching? $\left|\frac{ds}{dt}\right| = \boxed{\frac{17}{4}}$

KC

Related Rates

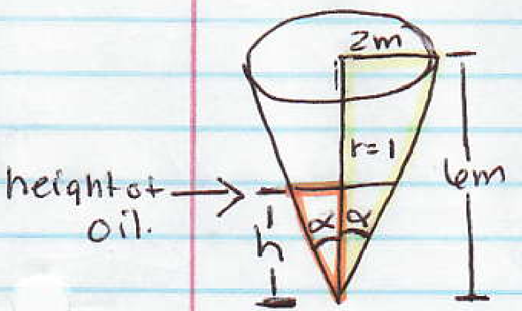
Summary:

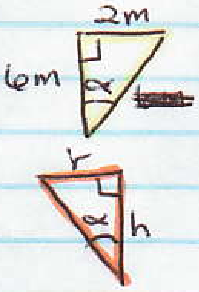
- Draw picture(s)
  - Numbers for constants
  - Variables for what changes over time.
- Write equations that are always true (geometry).
- Differentiate with respect to time.
- Solve for the requested rate of change.

Ex 2. An oil storage tank is built in the form of an inverted right circular cone with a height of 6m and a base radius of 2m. Oil is being pumped into the tank at a rate of 2 liters/min. $= 0.002\ m^3/min$ (since $1m^3 = 1000$ liters). How fast is the level of the oil rising when the tank is filled to a height of 3m?

* Some geometry formulas are on page 115 of your book. *

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$





$$\tan\alpha = \frac{2m}{6m}$$

$$\tan\alpha = \frac{r}{h}$$

$$\frac{r}{h} \times \frac{2}{6}$$

$$\frac{2h}{2} = \frac{6r}{2}$$

$$h = 3r \rightarrow r = \frac{h}{3}$$

$$\frac{dV}{dt} = 0.002\frac{m^3}{min}$$

$$\frac{dh}{dt} = ?\text{ when } h = 3m$$

$$(3^3 = 9 \cdot 3 = 27)$$

$$V = \frac{1}{3}\pi r^2 h \qquad V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = \frac{\pi h^3}{27}$$

$$\frac{dV}{dt} = \frac{d\left(\frac{\pi h^3}{27}\right)}{dt} = \frac{\frac{\pi}{27}\cdot 3h^2\,dh}{dt}$$

$$\left(\frac{\pi}{9}\cdot 9 = \pi\right)$$

$$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt} \qquad .002 = \frac{\pi}{9}(3)^2\frac{dh}{dt}$$

$$\frac{.002}{\pi} = \frac{\pi\frac{dh}{dt}}{\pi}$$

$$\frac{dh}{dt} = \frac{0.002}{\pi}$$