Calculus I 10/6/10

$\lim_{x \to 0} = \frac{\sin x}{x} = \lim_{x \to 0} \frac{\sin x - \sin 0}{x - 0}$

$\sin(0) = 0$

$= f'(0) = \underline{1}$

$f(x) = \sin(x)$ ..

$f'(x) = \cos(x)$

$f'(0) = \cos(0)$

$= \underline{1}$

check....

$= \frac{\sin(-.003)}{-0.003}$ ← do it with radians !.... in calculator

$= 0.9999985 00001$....

pretty close to $\underline{1}$ from the $\frac{\sin x - \sin 0}{x - 0} = \underline{1}$

tutoring
mon - after 2:20 coworth hall
E-mail - betosalinasjr@hotmail.com 2nd (205)
or go to learning center (humberto salinas)

One sided limits

Input value can get infinitely close but only to one side

Right hand limit $\lim_{x \to c^{+}}$ From right $x \approx c$ $x > c$

limits from the Right. then st $f(x) \approx L$

Equivalent $= \varepsilon + \approx \rightarrow$ st $(f(c + \varepsilon)) = L$

$\lim_{y \to -1}$ 2-sided

$$= \frac{1+y}{e^y - e^{-1}} = \frac{y-(-1)}{e^y - e^{-1}}$$

$$= \lim_{y \to -1} \left(\frac{e^y - e^{-1}}{y-(-1)} \right)^{-1}$$

notice that the limit $\lim_{y \to -1} \frac{e^y - e^{-1}}{y-(-1)} = f'(-1)$ where

$f(y) = e^y \quad f'(y) = e^y \quad f'(-1) = e^{-1}$

$$\lim_{y \to -1} \left(\frac{e^y - e^{-1}}{y-(-1)} \right)^{-1} = f((-1))^{-1} = (e^{-1})^{-1} = e^{(-1)(-1)}$$

$$= e^1 = e$$

Sanity Check:

try $y = -1.005$

$$\frac{1 + -1.005}{e^{-1.005} - e^{-1}} = \underbrace{2.7}_{\checkmark}250831\,9013$$

$$e = \underbrace{2.7}_{}1828182846$$

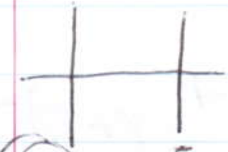
left hand Example

$$\lim_{x \to 5^-} \frac{\sqrt{x-1} - 2}{\sqrt{5-x}}$$

Equivalent: $\left(0 < \varepsilon \approx 0 \rightarrow st(f(c-\varepsilon)) = L \right.$

because you want to subtract a pos ε to get to the left

$$st\ \frac{\sqrt{5-\varepsilon-1}-2}{\sqrt{5-\varepsilon}} = st\left(\frac{\sqrt{4-\varepsilon}-2}{\sqrt{\varepsilon}}\right) \leftarrow \varepsilon > 0$$



$(5-\varepsilon) \approx 5$

$$= st\left(\frac{(\sqrt{4-\varepsilon}-2)(\sqrt{4-\varepsilon}+2)}{\sqrt{\varepsilon}(\sqrt{4-\varepsilon}+2)}\right)$$

$$= st\left(\frac{\sqrt{4-\varepsilon}^{\,2}-2^2}{\sqrt{\varepsilon}\,(4-\varepsilon+2)}\right)$$

$$\boxed{\varepsilon = \sqrt{\varepsilon}^{\,2} = \sqrt{\varepsilon}\cdot\sqrt{\varepsilon}}$$

$$st\left(\frac{\cancel{4}-\varepsilon-\cancel{4}}{\sqrt{\varepsilon}\cdot\sqrt{4-\varepsilon}+2}\right)$$

$$st\left(\frac{-\varepsilon}{\sqrt{\varepsilon}\sqrt{4-\varepsilon}+2}\right) \qquad \varepsilon = (\sqrt{\varepsilon}\cdot\sqrt{\varepsilon})$$

$$= st\left(\frac{\sqrt{\varepsilon}\cdot\cancel{\sqrt{\varepsilon}}}{\cancel{\sqrt{\varepsilon}}\sqrt{4-\varepsilon}+2}\right)$$

$$st\left(\frac{-\sqrt{\varepsilon}}{\sqrt{4-\varepsilon}+2}\right)$$

$$= \frac{-\sqrt{0}}{\sqrt{4-0}+2} = \frac{0}{\sqrt{6}} = \boxed{0}$$

$x \to 0 \quad = st(x + \varepsilon)$

x

$st\sqrt{\varepsilon} \leftarrow \sqrt{\text{infinitesimal}}$

$\downarrow \quad = \text{infinitesimal}$

$= 0$ ← this from the Right only....
if you approach from the left
it doesn't exist.

Recall $\lim\limits_{x \to 0} = \frac{|x|}{x}$ DNE

~~there~~ the answer was different from different sides.

if we approach from the right....

$$\lim_{x \to 0^{+}} \frac{|x|}{x} = st\left(\frac{|0+\varepsilon|}{0+\varepsilon}\right) = st\left(\frac{|\varepsilon|}{\varepsilon}\right)$$

$$= st\left(\frac{\varepsilon}{\varepsilon}\right) = st(1) = \underline{1}$$

From the Right

Left hand Limits

(from the left) : $\lim\limits_{x \to c^{-}} f(x) = L$

$(x \approx c \quad x < c) \Rightarrow f(x) \approx L$

Equivalent: $0 > \varepsilon \approx 0 \rightarrow f(c - \varepsilon) \approx L$