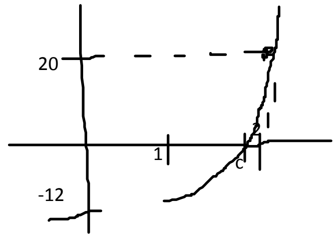
Properties of Continuous Functions 3.8

Tuesday, October 12, 2010
1:30 PM

1) IVT
2) EVT

1) $f(x) = x^5 + x - 14$
   $f(1) = 1^5 + 1 - 14 = -12$
   $f(2) = 2^5 + 1 - 14 = 20$

   $f(1) < 0 < f(2)$
   Is there some C where: $1 < C < 2$
   $f(c) = 0$
   Probably

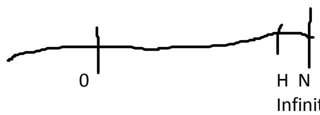


X	Fx
1	-12
1.1	-11.28
1.2	-10.31
1.3	-8.98
1.4	-7.22
1.5	-4.9
1.6	-1.91
1.7	+1.89
1.8	6.69
1.9	12.66
2	20

To prove C exists, use continuity of f(x) and use a infinite table of values

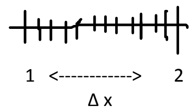
For every real x, there is an integer n< x
(Like $4>\pi$)
Transfer: for every hyper real x, there is a hyper integer n>x.

So, if H is an infinite hyper real, there is a hyper integer N>H, and so if H>0, then N>h>0, so N is infinite too.



Our example finite list can be written as:

X	F(x)
$X_0=1$	$f(x_0) = -12$
$X_1=1+\Delta x = 1.1$	$f(x_1) = -11.28$
$\Delta x = 2-1/10 = 0.1$	$f(x_2) = -10.31$
$X_3 = 1+3\Delta x = 1.3$	$f(x_3) = -8.98$..
….	….
$X_9=1+9\Delta x = 1.9$	$f(x_9) = 12.66$
$X_{10}=1+10\Delta x = 2$	$f(x_{10}) = 20$



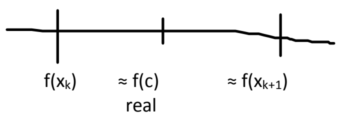
Let N be an infinite positive hyper integer and now let $\Delta x = (2-1)/N$ ($N \approx 0$)

For any finite table of values of f(x) from x=1 to x=2 since f(2) is not negative and f(1) is negative there must be a <u>last x-value</u> x_k on the list where $f(x_k) < 0$

By transfer, our infinite list also has a last x-value x_k where $f(x_k)<0$. So, $f(x_k)<0\leq f(x_k+1)$

Now use continuity of f(x) on [1,2]: let $c= st(x_k)$.
Since $x_{k+1} \approx x_k$. So, $st(x_{k+1}) = C$, $f(c) \approx f(x_k)$ & $f(c) \approx x_{k+1}$ because $C= st(x_k)$ & $C= st(x_{k+1}) \Rightarrow (C \approx x_k$ & $C \approx x_{k+1})$.

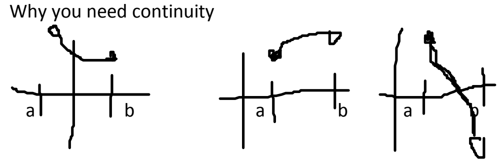
- $f(x_k)$ is a negative hyper real
- $f(x_{k+1})$ is a positive hyper real
- Both $f(x_k)$ & $f(x_{k+1})$ are infinitely close to the real f(c)

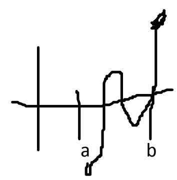


So, f(x) must be 0.

Intermediate Value Theorem (IVT)
If f(x) is continuous on [a,b], and f(a)<0<f(b) or f(a)>0>f(b) then f(c) = 0 for some C in (a,b)

Why you need continuity





Prove x= cos x has a solution
Let f(x) = cos x - x
Then f(x) = 0 is the same as x=cosx.
Also, f is continuous everywhere.
$f(0) = (\cos 0) - 0 = 1 - 0 = 1$
$f(3\pi) = (\cos 3\pi) - 3\pi = -1-3\pi$

By the IVT f(c) = 0 for some c in $(1,3\pi)$ so C=cos c for some c in $(1,3\pi)$