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Definition: c is a <u>critical point</u> of $f(x)$ if $f'(c)=0$ or if $f'(c)$ does not exist

critical points and end points are the usual places for maximums and minimums to occur.

How to find the max. and min. values of $f(x)$ <u>over $[a,b]$</u> <u>when f is continuous on $[a,b]$</u>.

step 0: check that f is really continuous on $[a,b]$
* the EVT ~~[illegible]~~ needs cont. to guarantee that the max./min. exist

step 1: find all critical points c in (a,b)

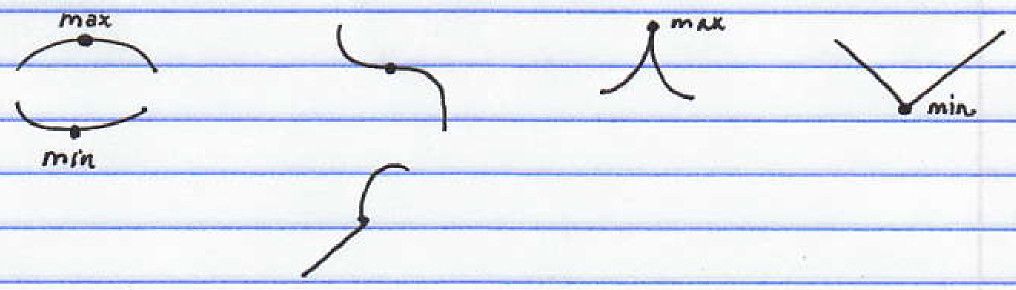
step 2: make a list of values: $f(a)$, $f(b)$, and $f(c)$ for all c's you've found

step 3: the greatest value is the max. value of $f(x)$ over $[a,b]$ – the least value on the list is the min. value of $f(x)$ over $[a,b]$.

Why does this work? See 3.5 & pictures:

critical pts.

$f'(c)=0$ $\quad\quad$ $f'(c)$ DNE



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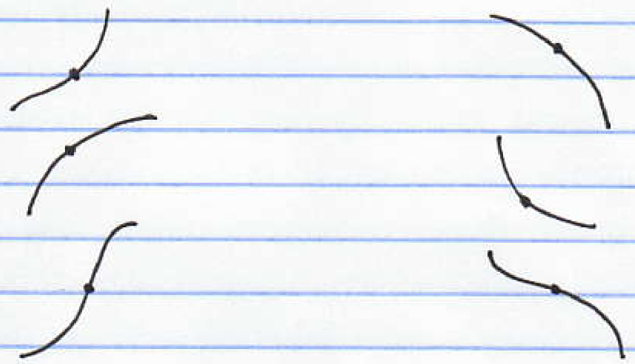
c not critical, not endpoint

$f'(c) > 0$ $\qquad\qquad$ $f'(c) < 0$



* never min or max of a noncritical point in the interior

Example: find the max. & min. values of $f(x) = x^3 - x$ over $[-1, 2]$

step 0: f is cts. everywhere

Step 1: $f'(x) = 3x^2 - 1$ exists everywhere

* could it be 0? $\quad 0 = 3x^2 - 1 \iff 1 = 3x^2 \iff \frac{1}{3} = x^2 \iff \pm\frac{1}{\sqrt{3}} = x$

$\pm\frac{1}{\sqrt{3}} = \pm 0.58\ldots$ both in $(-1, 2)$

step 2: $f(-1) = (-1)^3 - (-1) = -1 + 1 = 0$

$f(2) = 2^3 - 2 = 8 - 2 = 6$ $\quad\leftarrow$ max

$f\left(\frac{1}{\sqrt{3}}\right) = \left(\frac{1}{\sqrt{3}}\right)^3 - \frac{1}{\sqrt{3}} = \frac{1}{3\sqrt{3}} - \frac{1}{\sqrt{3}} = -0.3849\ldots$ $\quad\leftarrow$ min

$f\left(-\frac{1}{\sqrt{3}}\right) = \left(\frac{-1}{\sqrt{3}}\right)^3 - \left(\frac{-1}{\sqrt{3}}\right) = \frac{-1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = 0.3849\ldots$

The max value of $x^3 - x$ over $[-1, 2]$ is 6 and min. value of $x^3 - x$ over $[-1, 2]$ is $\frac{1}{3\sqrt{3}} - \frac{1}{\sqrt{3}}$