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Alternative way to find just the maximum or minimum value of a function $f(x)$ over an interval I.

gives minimum information

(0) check for continuity on I.
(Probably won't work without It.)

(1) Find the critical points within the interior of Interval of I.

(2) If you find none or more than one, use "closed interval" method. (method from last 2 days)

(3) If there's exactly one critical point in the interval I, use the <u>direct test</u> or the 2nd <u>Derivative Test</u> to determine if $f(c)$ is the max value (or minimum) you're looking for.

(4) If $f(c)$ is not the max or min value you're looking for, then use the "Closed Interval method"

<u>Direct Test</u> Pick 2 numbers in function, (p, q), with $p < c < q$

If:

$f(p) > f(c) < f(q) \Rightarrow f(c)$ is min.

$f(p) < f(c) > f(q) \Rightarrow f(c)$ is max.

$f(p) < f(c) < f(q) \Rightarrow f(c)$ is neither

$f(p) > f(c) > f(q) \Rightarrow f(c)$ is neither

2^{nd} Derivative Test

$\cup$ $f''(c) > 0 \Rightarrow f(c)$ is min.

$\cap$ $f''(c) < 0 \Rightarrow f(c)$ is max.

? $f''(c) = 0 \Rightarrow$ Use Direct Test

? $f''(c)$ DNE $\Rightarrow$ Use Direct Test

Ex. 1

Find the min. value of $\frac{30}{x} + 2x$ over $(0, \infty)$

(0) $f(x) = \frac{30}{x} + 2x$ is continuous everywhere except at 0.

(1) $f'(x) = -\frac{30}{x^2} + 2$ $\quad$ $f'(0)$ DNE, but not in interval

solve $\quad 0 = -\frac{30}{x^2} + 2 \rightarrow \frac{30}{x^2} = 2 \rightarrow 30 = 2x^2 \rightarrow 15 = x^2$

$\Downarrow$

$\sqrt{15} = x$

$\sqrt{15}$ is in $(0, \infty)$

(3) $f'(x) = -30x^{-2} + 2 \rightarrow f''(x) = -30(-2)x^{-2-1} + 0$

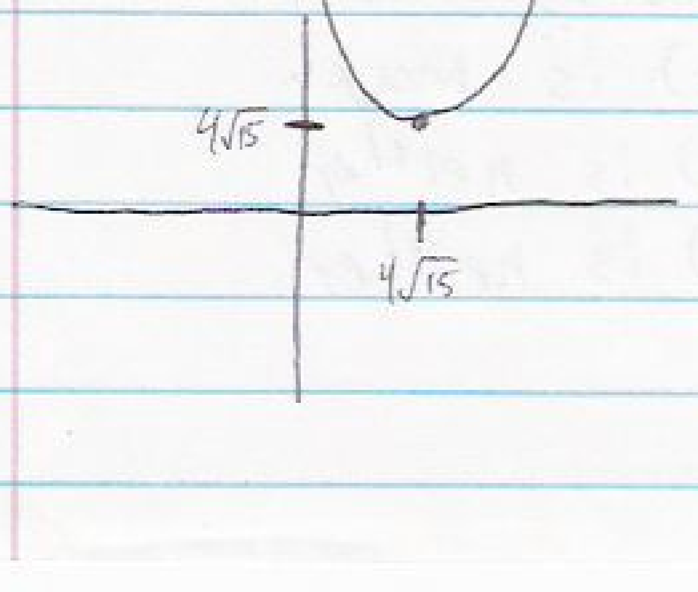
$\Downarrow$

$60x^{-3} = \frac{60}{x^3}$

$f''(\sqrt{15}) = \frac{60}{(\sqrt{15})^3} > 0$

$\Downarrow$

$f(\sqrt{15}) = \frac{30}{\sqrt{15}} + 2\sqrt{15}$ is the min.



simplify

$30 = 2 \cdot 15 = 2\sqrt{15}\sqrt{15} \Rightarrow f(\sqrt{15}) = 2\sqrt{15} + 2\sqrt{15}$

$\Downarrow$

$\boxed{4\sqrt{15}}$

ex. 2

Find the max. value of $2x-e^{3x}+1$ over $[0,\infty)$

(0) $f(x) = 2x - e^{3x}+1$ is continous everywher

(1) $f'(x) = 2 - e^{3x}(3)+0 = 2-3e^{3x}$

$f'(x)$ exist everywhere

solve $0 = 2-3e^{3x}$

$3e^{3x}=2$

$e^{3x}=2/3$

$3x=\ln(2/3)$

$x=\frac{1}{3}\ln(2/3)$

less than zero, No critical points in $[0,\infty)$

$[0,\infty) \Rightarrow [0,H]$ where $H>0$ & H infinite

$f(0) = 2(0) - e^{3(0)}+1 = 0-1+1=0$

$f(H) = 2H - e^{3H}+1 =$ infinetly ~~positive~~ negative

No min. & max value $=0$

ex. 3

Find max. value of $x^2(1-x)$ over $(0,1)$

$f(x) = x^2(1-x)$ is continous everywhere

~~$f'(x) = 2x$~~ $f(x) = x^2-x^3 \Rightarrow f'(x) = 2x-3x^2$

$f'(x)$ exist everywhere

solve $0 = 2x-3x^2 \Rightarrow x(2-3x)$

~~$x(\quad)(\quad)$~~ $x=0$ not in interval

$x=\frac{2}{3}$ is in interval

continue

Direct Test

pick $\underbrace{\frac{1}{2} < \frac{2}{3} < \frac{3}{4}}_{\text{all in interval}}$

$f(\frac{1}{2}) = \frac{1}{4}(1-\frac{1}{2}) = \frac{1}{8} = 0.125$

$f(\frac{2}{3}) = \frac{4}{9}(1-\frac{2}{3}) = \frac{4}{27} = 0.15$

$f(\frac{3}{4}) = \frac{9}{16}(1-\frac{3}{4}) = \frac{9}{64} = 0.14$