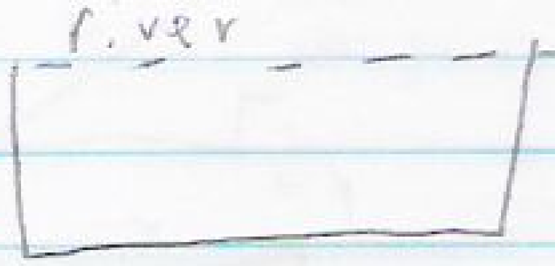
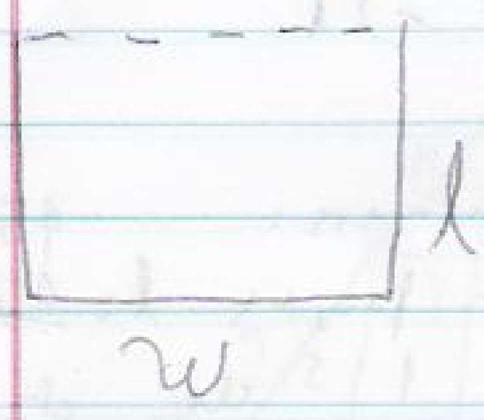
Today: more optimization examples (3.5)

Tomorrow: review
Wensday: Midterm II



A farmer wants to maximize the area inside a three-sided fence enclosing a rectangle on the (straight) shore of a river. He has 18,000 feet of fence. What are the dimensions of the optimal fence?



$A = l * w$
Maximize A

measuring l, w in feet.

(3) $18,000 = 2L + w$

(4) $18000 - 2L = W \Rightarrow A = l(18000 - 2L)$

$A = 18000L - 2L^2$

(5) Solve $\frac{dA}{dl} = 0$

$A = 18000L - 2l^2$

$\frac{dA}{dl} = 18000 - 4L$

$0 = 18000 - 4L$

$\frac{18000}{4} = l = 4,500$

6. Test that the max really is at $l = 4500$.

Direct Test: pick p, q, with $p < 4500 < q$

Plug p, 4500, q into A.

Check that A is greatest at $l = 4500$

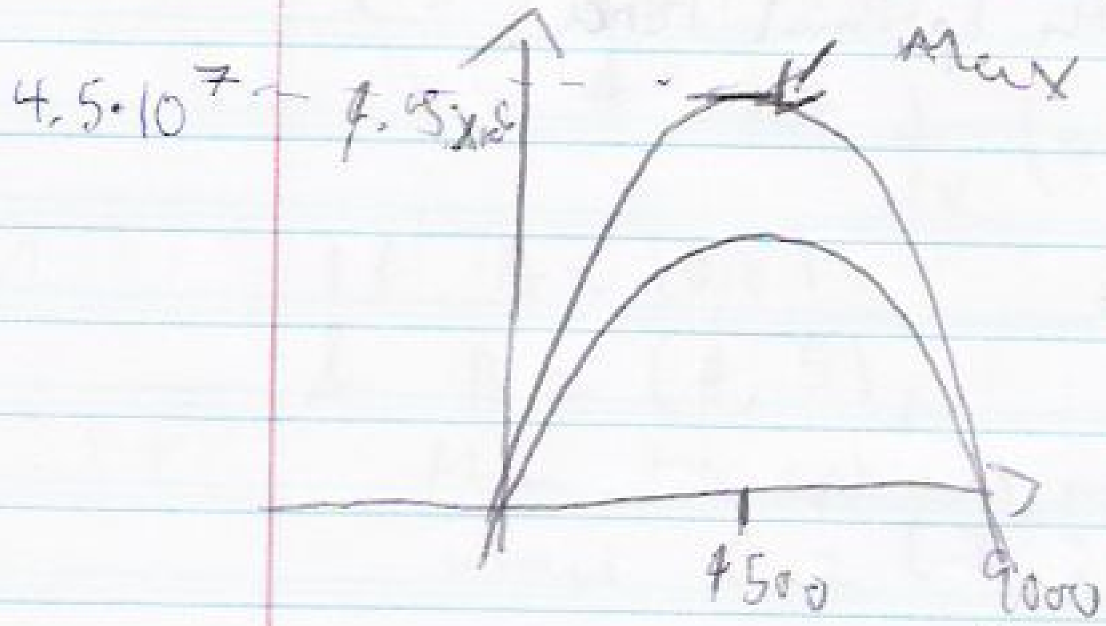
Direct test:

$p = 0 < 4500 < q = 9000$

$l = 0 \Rightarrow A = l(18000 - 2l) = 0$

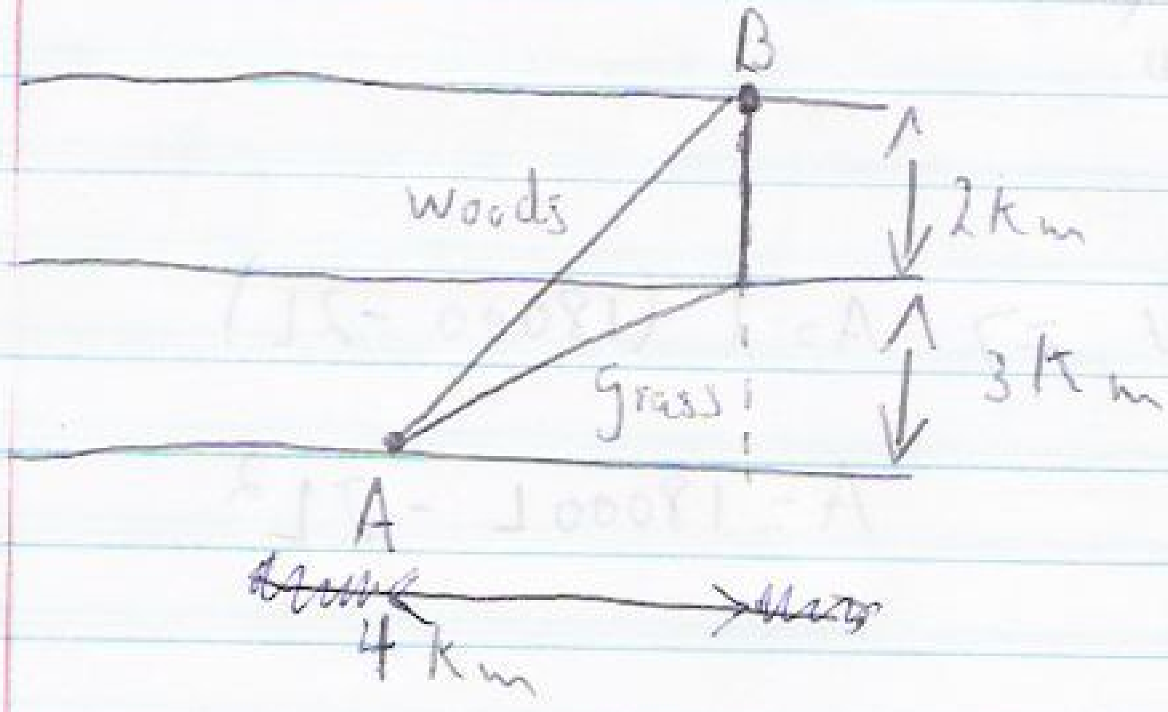
$l = 4500 \Rightarrow A = 4500(18000 - 2(4500)) = 40{,}500{,}000$

$l = 9000 \Rightarrow A = 9000(18000 - 2(9000)) = 0$



$A = 18000l - 2l^2$

Final answers: 2 sides of length $l = \cancel{9000}\ 4500$ ft each and 1 side of length $w = 18{,}000 - 2l = 9000$ ft



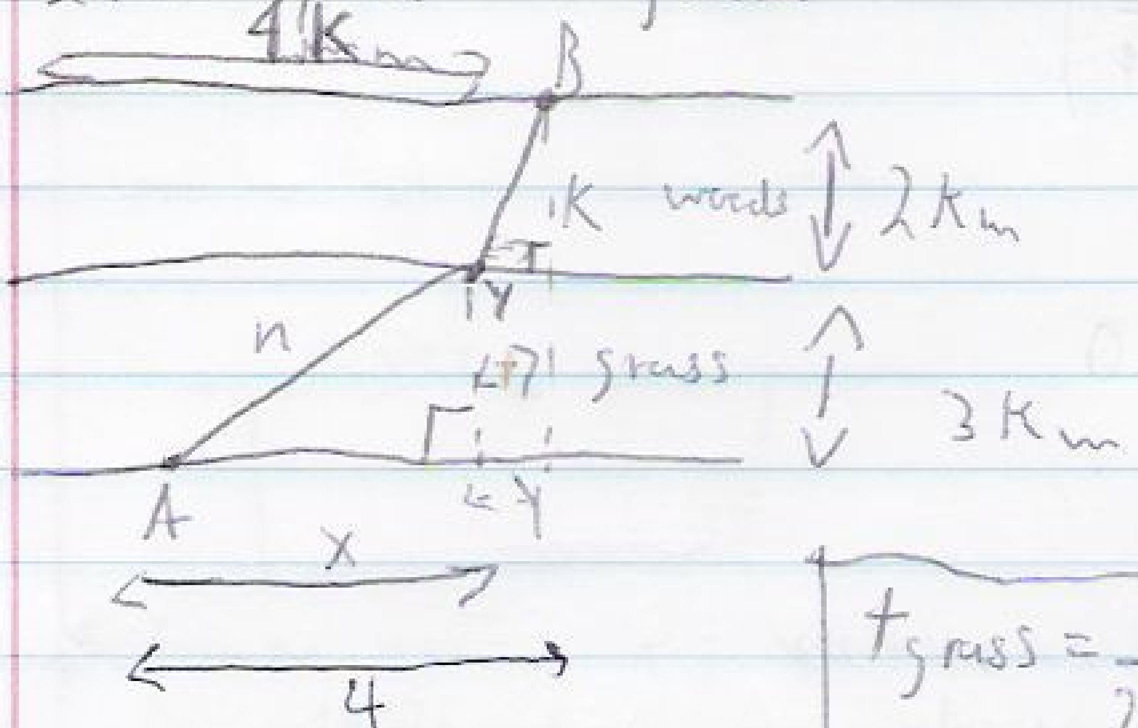
If you can walk twice as fast through the grass find the quickest route from A to B.

(average) speed = $\dfrac{\text{distance}}{\text{time}}$

woods = $V = \dfrac{k}{t_{woods}}$, ~~t_{woods}~~ $t_{woods} = \dfrac{k}{V}$

V = speed in woods

2V = speed in grass.

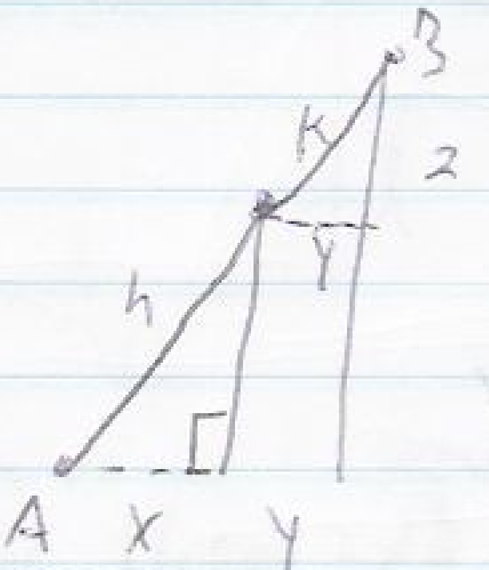


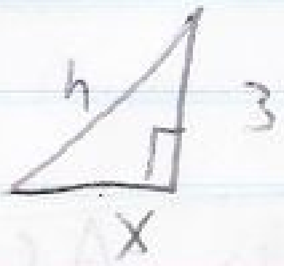
time = $\dfrac{\text{distance}}{\text{speed.}}$

grass time = $\dfrac{h}{2V}$

$$\boxed{t_{grass} = \frac{h}{2v} \ \& \ t_{woods} = \frac{k}{V}}$$

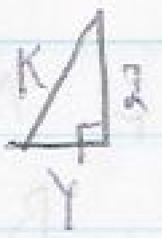
Travel time $t = t_{grass} + t_{woods} = \dfrac{h}{2v} + \dfrac{k}{v} = \dfrac{1}{2v}(h+2k)$





$x^2 + 3^2 = h^2$

$h = \sqrt{x^2+9}$



$y^2 + 2^2 = k^2$

$\sqrt{y^2+4} = k$

$$\boxed{t = \frac{1}{2v}\left(\sqrt{x^2+9} + 2\left(\sqrt{y^2+4}\right)\right) \quad \text{Minimize } t}$$

Constraint

$x + y = 4$

$y = 4 - x$

$t = \left(\dfrac{1}{2v}\left(\sqrt{x^2+9} + 2\left(\sqrt{y^2+4}\right)\right)\right.$

$\dfrac{dt}{dv} = \dfrac{1}{2v}\left(\dfrac{1}{2}(x^2+9)^{-1/2}(x^2+9)'\right.$ $\cancel{+ 2 \cdot \frac{k}{2}(y^2+4)^{-1/2}(y^2+4)'}$

$+ 2 \cdot \dfrac{1}{2}\left((4-x)^2+4\right)^{-1/2} \cdot \left((4-x)^2+4\right)'$

$2(4-x)^{2-1}(4-x)'+0$

$$\frac{dt}{dv} = \frac{1}{2v}\left(\frac{1}{2\sqrt{x^2+4}}\cdot 2x + 2\,\frac{1}{\sqrt{(4-x)^2+4}}\cdot 4-x-1\right)^2$$

$$\frac{dt}{dv} = \frac{1}{2v}\left(\frac{x}{\sqrt{x^2+4}} + \frac{2x-8}{\sqrt{(4-x)^2+4}}\right)$$

Find where this equals 0

Estimate

$x \approx 3.214557$

Use Direct test

$x=0 = t = \frac{1}{2v}(11.944\ldots\ km)$

$x = 3.214557 \quad t = \frac{1}{2v}(8.6943\ldots\ km)$

$x = 4 \quad t = \frac{1}{2v}(9km)$

If $A=(0,0)$

& $B=(4,5)$,

then the optimal part is ACB,

where $C = (3.214554\ldots, 3)$