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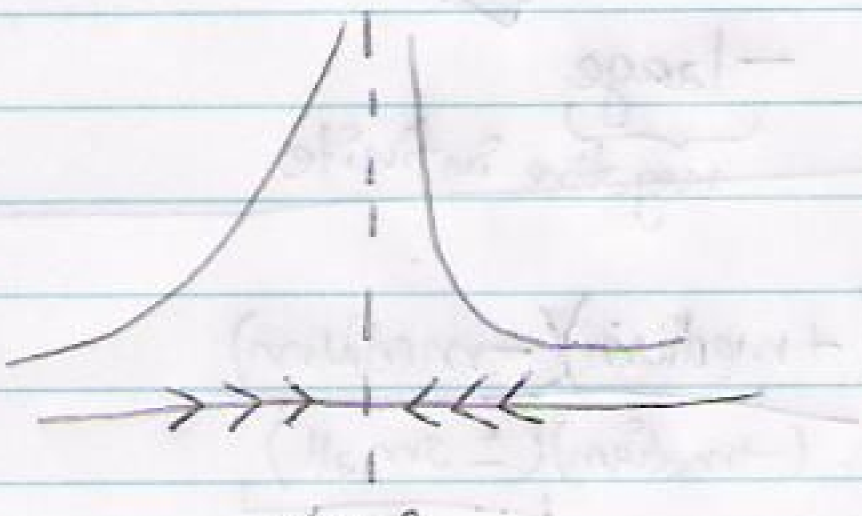
10/28/10

Infinite Limits (5.1)

$\lim_{x \to c} f(x) = \infty$ means

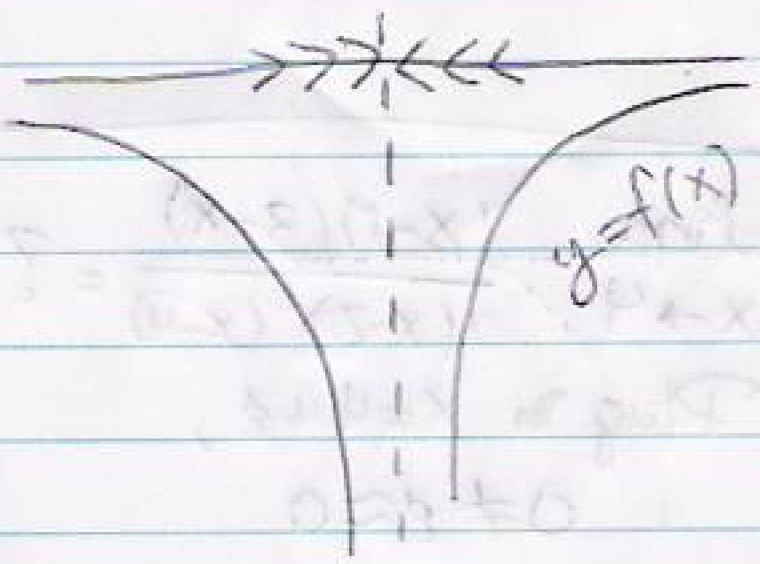
$f(c+\varepsilon)$ is a positive infinite hyperreal for all nonzero infinitesimal ε.

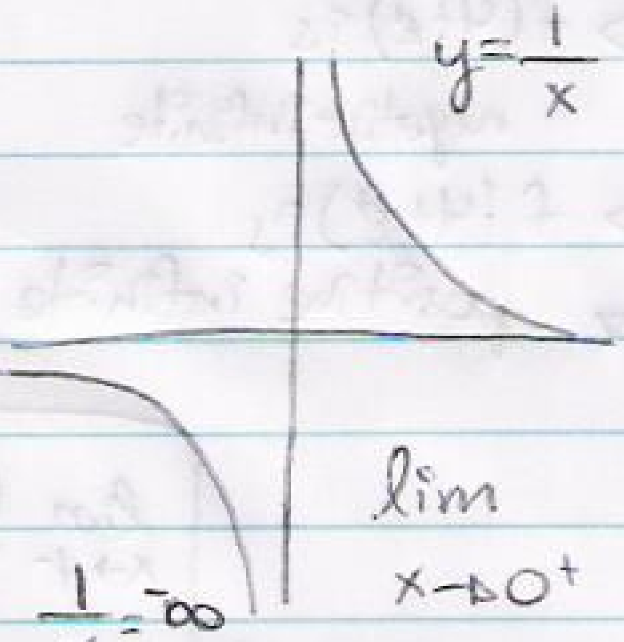
A useful description of how $\lim_{x \to c} f(x)$ fails to exist (as a number).



$x = c$

$\lim_{x = c} f(x) = -\infty$





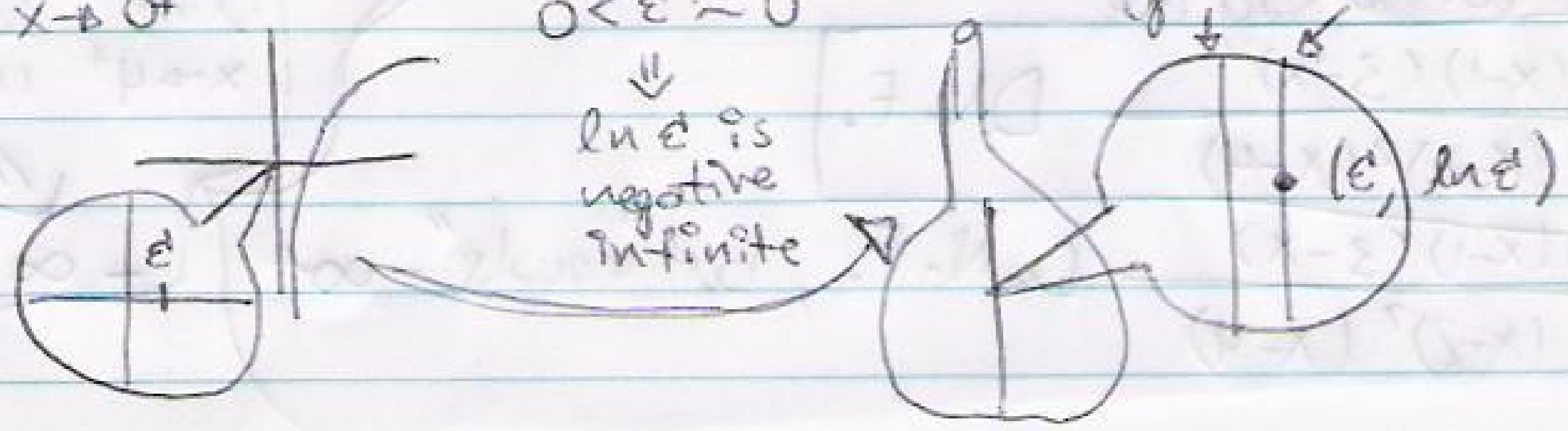
$\lim_{x \to 0^-} \frac{1}{x} = -\infty$ $\qquad$ $\lim_{x \to 0^+} \frac{1}{x} = \infty$

$\lim_{x \to 0^+} \ln x = -\infty$

$0 < \varepsilon \approx 0$

$\Downarrow$

$\ln \varepsilon$ is negative infinite

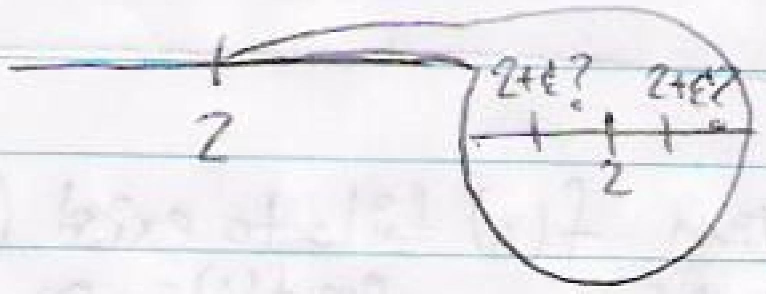


$$f(x) = \frac{(x-1)(3-x)}{(x-2)^2(x-4)}$$

$$\lim_{x\to 2} f(x) \overset{?}{=} \boxed{-\infty}$$

Plug in $x = 2+\varepsilon$

where $0 \neq \varepsilon \approx 0$



$$f(2+\varepsilon) = \frac{(2+\varepsilon-1)(3-(2+\varepsilon))}{(2+\varepsilon-2)^2(2+\varepsilon-4)} = \frac{(1+\varepsilon)(1-\varepsilon)}{\varepsilon^2(\varepsilon-2)}$$

$1+\varepsilon \approx 1 \Rightarrow 1+\varepsilon > 0$

$1-\varepsilon \approx 1 \Rightarrow 1-\varepsilon > 0$

$\varepsilon^2 > 0$

$\varepsilon - 2 \approx -2 < 0$

medium → finite
~~non~~ infinitesimal

small → infinitesimal

large → infinite

Abeeviations

$$\underbrace{\frac{(+\text{medium})(+\text{medium})}{(+\text{small})(-\text{medium})}}_{\underbrace{-\text{large}}_{\text{negative infinite}}}$$

$$\lim_{x\to 4} \frac{(x-1)(3-x)}{(x-2)^2(x-4)} = ?$$

Plug in $x = 4+\varepsilon$,

$0 \neq \varepsilon \approx 0$

$$\frac{(4+\varepsilon-1)(3-(4+\varepsilon))}{(4+\varepsilon-2)^2(4+\varepsilon-4)}$$

$$\frac{\overbrace{(3+\varepsilon)}^{\approx 3 > 0}\overbrace{(-1-\varepsilon)}^{\approx -1 < 0}}{\underbrace{(2+\varepsilon)^2}_{\approx 2^2 = 4 > 0}\underbrace{\varepsilon}_{\text{could be positive or negative}}}$$

$$\frac{(+\text{medium})(-\text{medium})}{(+\text{medium})\underbrace{(\pm\text{small})}_{\varepsilon}}$$

$\varepsilon > 0 \Rightarrow f(4+\varepsilon)$ is negative infinite

$\varepsilon < 0 \Rightarrow f(4+\varepsilon)$ is positive infinite

$$\boxed{\lim_{x\to 4^-} \frac{(x-1)(3-x)}{(x-2)^2(x-4)} = +\infty}$$

$$f(x) = \frac{(x-1)(3-x)}{(x-2)^2(x-4)}$$

and

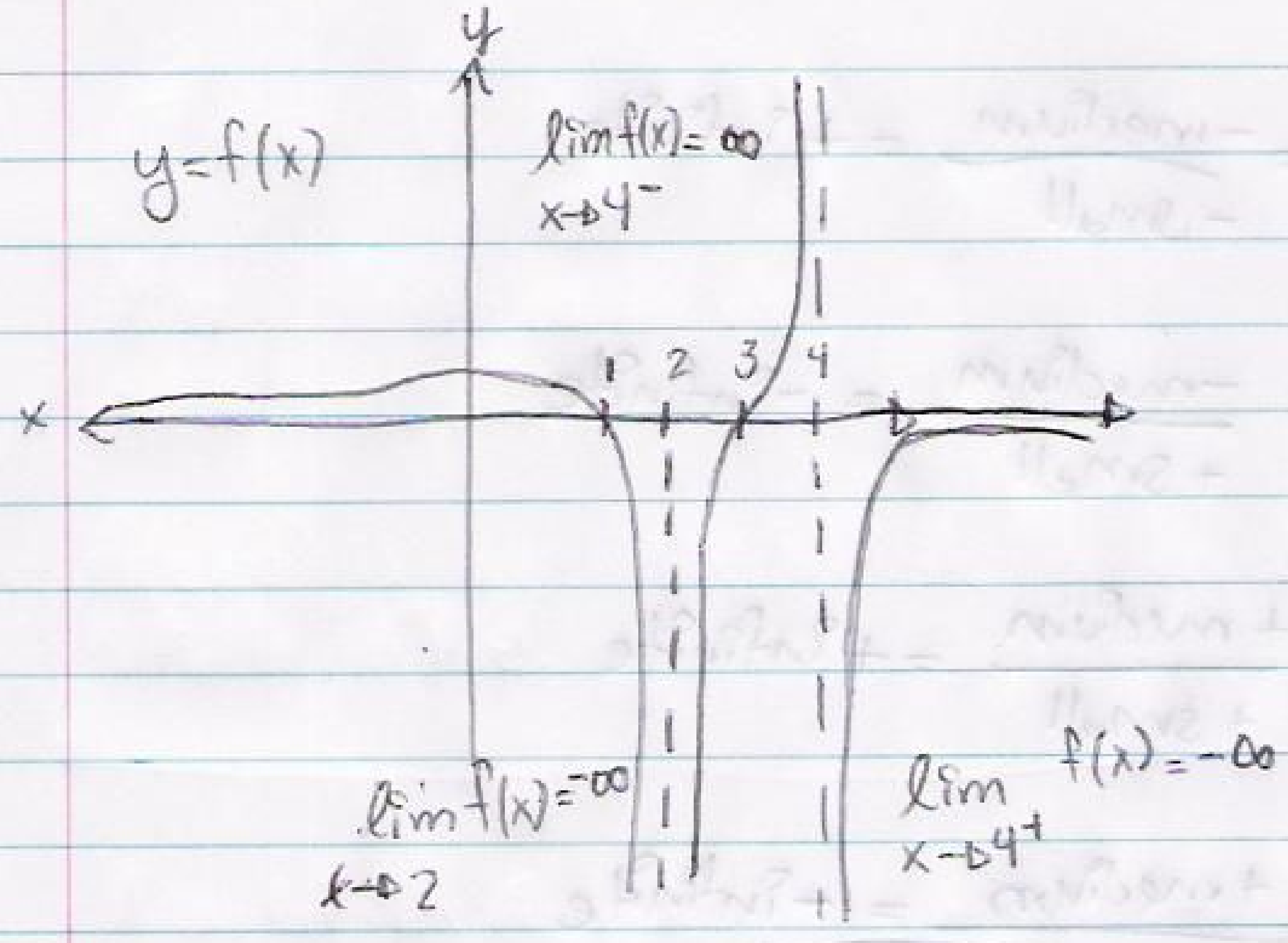
$$\boxed{\lim_{x\to 4^+} \frac{(x-1)(3-x)}{(x-2)^2(x-4)} = -\infty}$$

The most we can say is:

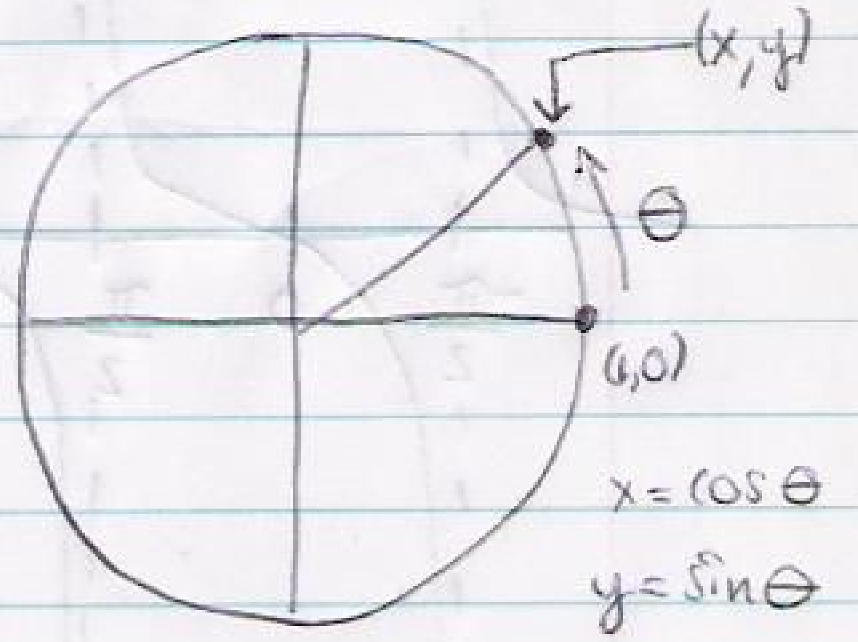
$$\boxed{\lim_{x\to 4} \frac{(x-1)(3-x)}{(x-2)^2(x-4)} \quad \text{D.N.E.}}$$

$$\lim_{x\to 4^+} \frac{(x-1)(3-x)}{(x-2)^2(x-4)} \quad \text{D.N.E., but "equals" } -\infty$$

$y=f(x)$



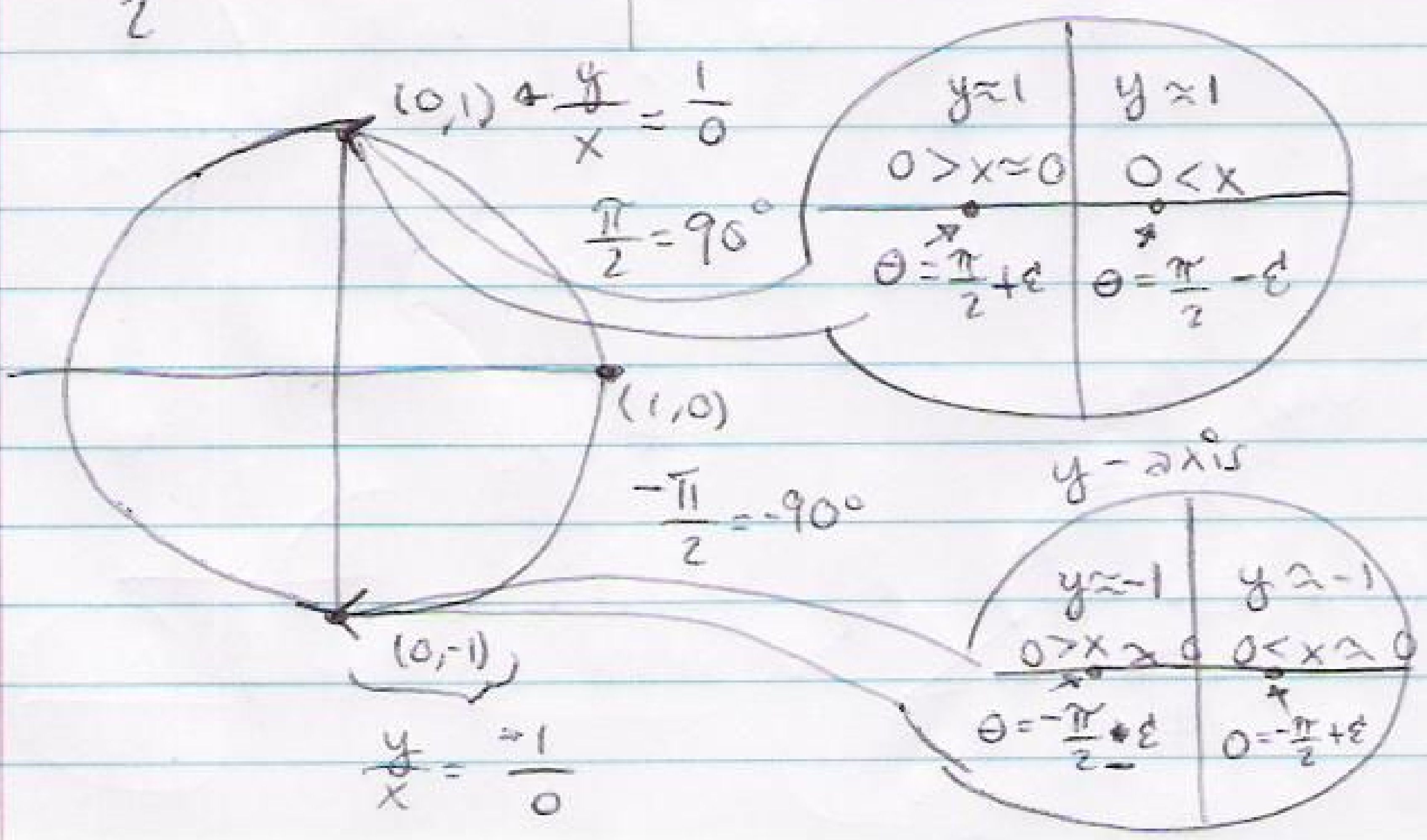
$\lim_{x\to -\frac{\pi}{2}^-} \tan x = \infty$	Plug in $-\frac{\pi}{2} - \varepsilon$
$\lim_{x\to -\frac{\pi}{2}^+} \tan x = -\infty$	Plug in $-\frac{\pi}{2} + \varepsilon$
$\lim_{x\to \frac{\pi}{2}^-} \tan x = \infty$	Plug in $\frac{\pi}{2} - \varepsilon$
$\lim_{x\to \frac{\pi}{2}^+} \tan x = -\infty$	Plug in $\frac{\pi}{2} + \varepsilon$



$x = \cos\theta$

$y = \sin\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$

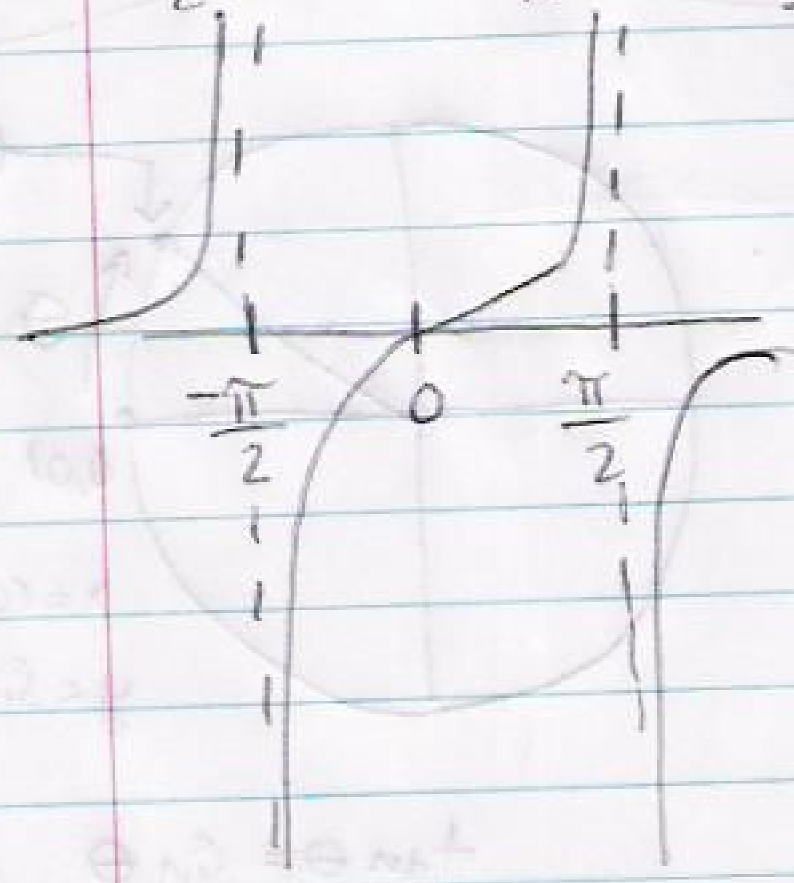


$$-\frac{\pi}{2} - \varepsilon = \frac{y}{x} = \frac{-\text{medium}}{-\text{small}} = +\text{infinite}$$

$$-\frac{\pi}{2} + \varepsilon = \frac{y}{x} = \frac{-\text{medium}}{+\text{small}} = -\text{infinite}$$

$$\frac{\pi}{2} - \varepsilon = \frac{y}{x} = \frac{+\text{medium}}{+\text{small}} = +\text{infinite}$$

$$\frac{\pi}{2} + \varepsilon = \frac{y}{x} = \frac{+\text{medium}}{-\text{small}} = -\text{infinite}$$



$y = \tan x$