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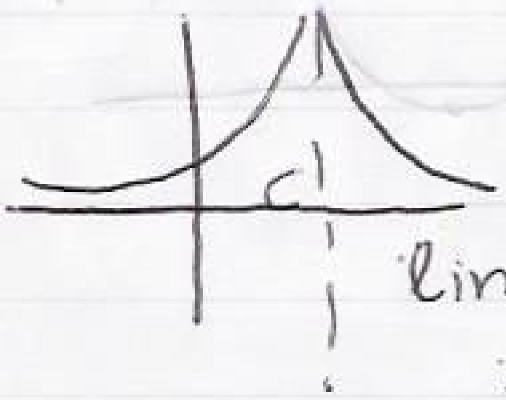
Limits at $\pm\infty$

Infinite limits:

Recall last time:



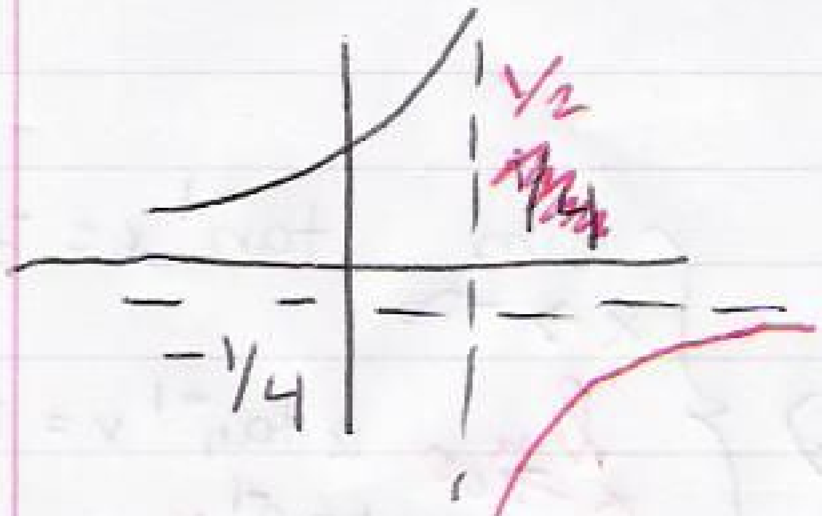
$\lim\limits_{x\to c} f(x) = \infty$

$\lim\limits_{x\to\infty} f(x) = L$ means for all positive infinite H, $st(f(H)) = L$

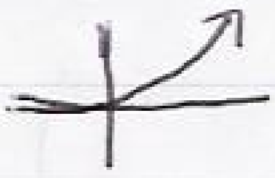
$\lim\limits_{x\to\infty} \dfrac{x+5}{2-4x} = ?$

Plug in x=h (H>0 H infinite)

$$\frac{H+5}{2-4H} = \frac{(H+5)/H}{(2-4H)/H} = \frac{1+5/H}{2/H-4} \approx \frac{1+0}{0-4} = -\frac{1}{4} \qquad \frac{x+5}{2-4x} \to -\frac{1}{4}$$

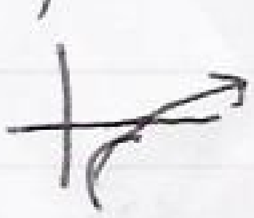


$\lim\limits_{x\to-\infty} e^x = 0$ means $e^{-H} \approx 0$ for all infinite positive H



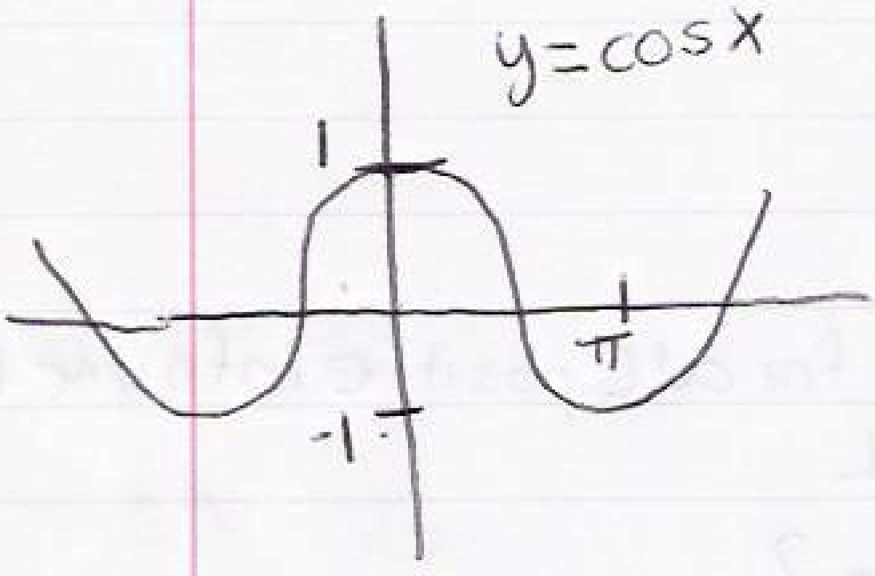
$\lim\limits_{x\to\infty} e^x = \infty$ means e^H is positive infinite for all positive infinite H.

$\lim\limits_{x\to\infty} \ln x = \infty$ means $\ln H$ is positive infinite for all positive infinite H

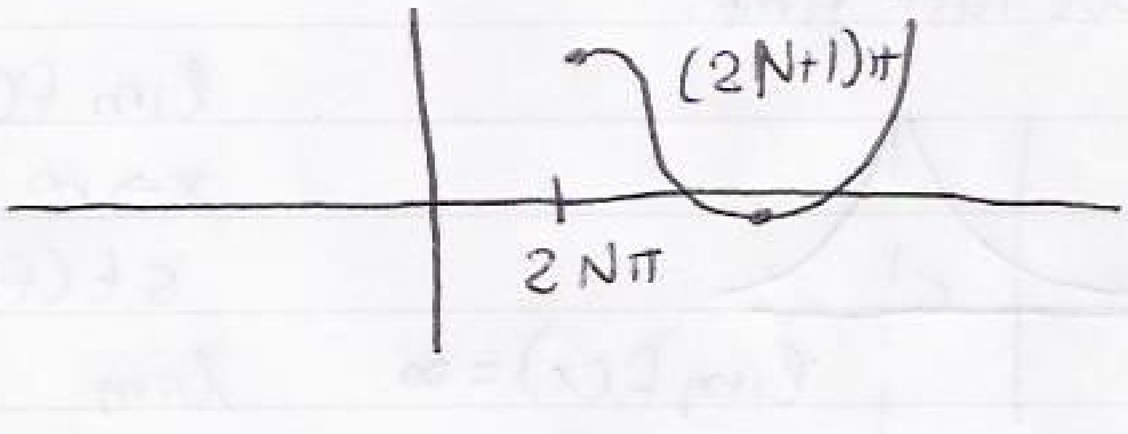


doesnt exist
not ∞ or $-\infty$

$$\lim_{x \to \infty} \cos x$$



Go infinitely far to the right Let N be positive infinite hyperinteger

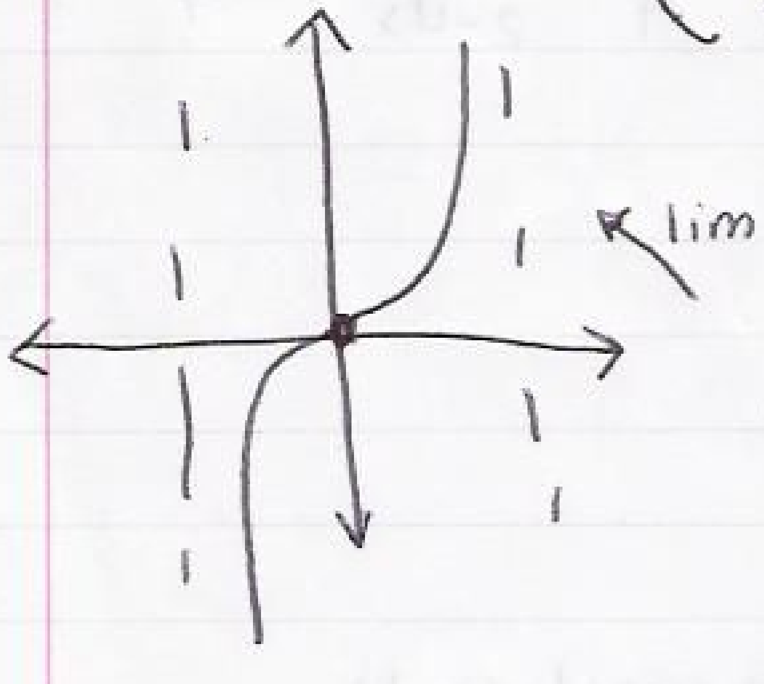


$y = \tan^{-1} x$ is defined by

$-\frac{\pi}{2} < y < \frac{\pi}{2}$

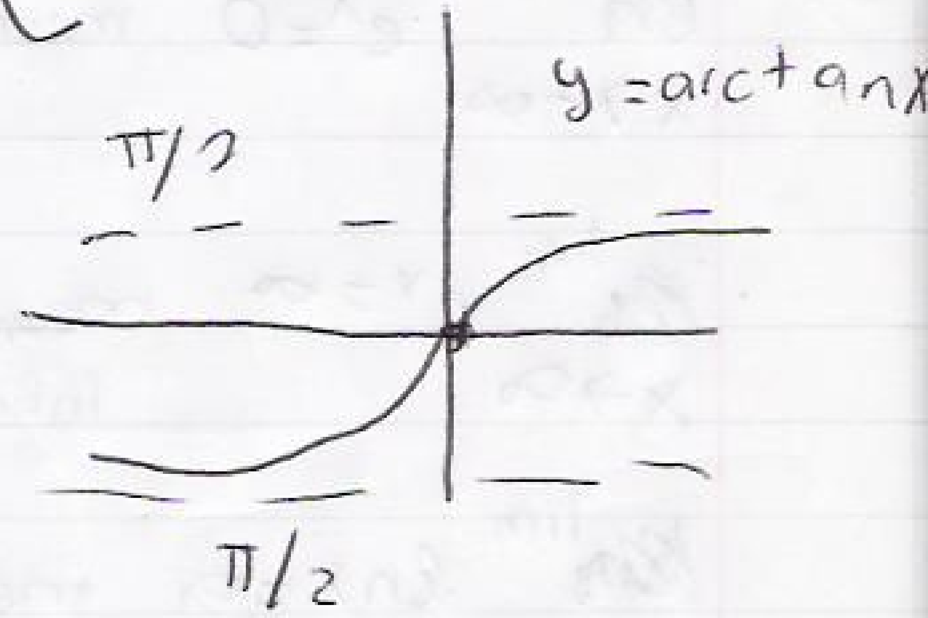
$x = \tan y$

$$\begin{cases} x = \tan y \\ -\frac{\pi}{2} < y < \frac{\pi}{2} \end{cases}$$



$$\left.\begin{aligned} &\lim_{y \to -\frac{\pi}{2}^+} \tan y = -\infty \\ &\lim_{y \to \frac{\pi}{2}^-} \tan y = \infty \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} &\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2} \\ &\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2} \end{aligned}\right.$$

$y = \tan^{-1} x$



$$\lim_{x \to \infty} = \frac{\sin x}{x} = ?$$ Plug in $x = H$ positive inf

$$0 \approx \frac{-1}{H} \leq \frac{\sin H}{H} \leq \frac{1}{H} \approx 0 \Rightarrow \frac{\sin H}{H} \approx 0$$

$$\lim_{x\to-\infty} (1-x^2+x^3)(x^5+8x^6+2x^7-6) = \infty :$$

$$x^3\,\frac{1-x^2+x^3}{x^3} \qquad x^7\,\frac{x^5+8x^6+2x^7-6}{x^7}$$

$$x^3\left(\frac{1}{x^3}-\frac{x^2}{x^3}+1\right) \qquad x^7\left(\frac{x^5}{x^7}+\frac{8x^6}{x^7}+2-\frac{6}{x^7}\right)$$

$$(-H)^3\left(\frac{1}{(-H)^3}-\frac{1}{-H}+1\right) \qquad \approx 0-0+1=1$$

$$\cdot(-H)^7\left(\frac{1}{(-H)^2}+\frac{8}{-H}+2\,\frac{-6}{-H^7}\right) \qquad \approx 0+0+2-0$$

$$(-H)^{10}\ \cancel{(1+\varepsilon)}\ \underbrace{(1+\varepsilon)(2+\delta)}_{\approx 2} \quad \varepsilon\approx 0 \quad \delta\approx 0 \qquad = H^{10}=(H^2)^5 \approx 2$$

$$((-H)^2)^5$$

$$\lim_{x\to\infty} (3-x^3+x^4-x^7) \qquad x^7\left(\frac{3}{x^7}-\frac{1}{x^4}+\frac{1}{x^3}-1\right) \qquad H^7\left(\frac{3}{H^7}-\frac{1}{H^4}+\frac{1}{H^3}-1\right)$$

$$\approx 0-0+0-1 = \boxed{-\infty}$$

$$\lim_{x\to\infty} (\sqrt{x^2+3}-x) = x\left(\frac{\sqrt{x^2+3}}{\sqrt{x}}-\frac{x}{x}\right) = x\left(\sqrt{\frac{x^2+3}{x^2}}-1\right) =$$

$$x\left(\sqrt{1+\frac{3}{x^2}}-1\right) = H\left(\sqrt{1+\frac{3}{H^2}}-1\right. \qquad \approx \cancel{2}\sqrt{1+0}\approx 0$$

$$= \left(\frac{1+3/H^2-1}{\sqrt{1+3/H^2}+1}\right) = H\left(\frac{3/H^2}{\sqrt{1+3/H^2+1}}\right) = \frac{3/H}{\sqrt{1+3/H^2}} \approx \frac{0}{\sqrt{1+0}+1}$$

$$\frac{0}{2}=0$$