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L'Hôpital's Rule $\quad \sin \pi = 0$

$$\lim_{x \to \pi} \frac{\sin x}{\ln x - \ln \pi}$$

$$= st\left(\frac{\sin(\pi+\varepsilon)}{\ln(\pi+\varepsilon)-\ln\pi}\right)$$

where of $\varepsilon \simeq 0$

$$= st\left(\frac{\sin(\pi+\varepsilon)-\sin\pi}{\ln(\pi+\varepsilon)-\ln\pi}\right) \quad \text{write } \Delta x = \varepsilon$$

$$= st\left(\frac{\sin(\pi+\Delta x)-\sin\pi}{\ln(\pi+\Delta x)-\ln\pi}\right) = st\left(\frac{(\sin(\pi+\Delta x)-\sin\pi)/\Delta x}{(\ln(\pi+\Delta x)-\ln\pi)/\Delta x}\right) =$$

$\sin(\pi+\Delta x)-\sin\pi \simeq \sin\pi - \sin\pi = 0$

$\ln(\pi+\Delta x)-\ln\pi \simeq \ln\pi - \ln\pi = 0$

$$\frac{st((\sin(\pi+\Delta x)-\sin\pi)/\Delta x)}{st((\ln(\pi+\Delta x)-\ln\pi)/\Delta x)} = \cancel{\frac{\sin'\pi}{\sin'\pi}}\ \frac{\sin'\pi}{\ln'\pi} = \frac{\cos\pi}{1/\pi} =$$

$$\pi\cos\pi = \pi(-1) = \boxed{-\pi}$$

$$\lim_{x\to\pi} \frac{\sin x}{x} = st\left(\frac{\sin(\pi+\Delta x)}{(\pi+\Delta x)}\right) = st\left(\frac{\sin(\pi+\Delta x)-\overbrace{\sin\pi}^{0}}{\pi+\Delta x\underbrace{}_{-\pi \text{ missing}}}\right) \neq \frac{\sin' x}{x'} \text{ at } x=\pi$$

$$\lim_{x\to 0} \frac{\sin x}{x} = st\left(\frac{\sin(0+\Delta x)}{(0+\Delta x)}\right) = st\left(\frac{\sin(0+\Delta x)-\sin 0}{(0+\Delta x)-0}\right) = \frac{\sin' x}{x'} \text{ at } x=0$$

If $\overbrace{\lim_{x\to c} f(x) = 0 = \lim_{x\to c} g(x)}^{0/0 \text{ form}}$ or $\overbrace{\lim_{x\to c} = \pm\infty \ \& \ \lim_{x\to c} g(x) = \pm\infty}^{\infty/\infty \text{ form}}$

and $\lim_{x\to c} \frac{f'(x)}{g'(x)}$ exists, or is $= \pm\infty$, then

$$\boxed{\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}}$$

$$\lim_{x\to\infty} \frac{\ln x^{\nearrow\infty}}{\sqrt{x}_{\searrow\infty}} = \lim_{x\to\infty} \frac{(\ln x)'}{(\sqrt{x})'} = \frac{(1/x)}{(1/2\sqrt{x})} \cdot \frac{2x\sqrt{x}}{2x\sqrt{x}} =$$

$$\lim_{x\to\infty} \frac{2\sqrt{x}}{\underset{x=(\sqrt{x})^2}{x}} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = st\left(\frac{2}{\sqrt{H}}\right) = 0$$

$\to H$ infinite positive

$$\lim_{x\to 0} \frac{x^2+3\sin x}{e^x-\cos x} \begin{array}{l} \to 0^2 + \overbrace{3\sin 0}^{0} = 0 \\ \to e^0 - \cos 0 = 1-1 = 0 \end{array}$$

$$= \lim_{x\to 0} \frac{(x^2+3\sin x)'}{(e^x-\cos x)'} = \lim_{x\to 0} \frac{2x+3\cos x}{e^x+\sin x} = \frac{2\cdot 0 + 3\overbrace{\cos 0}^{1}}{\underbrace{e^0}_{1} + \underbrace{\sin 0}_{0}} = 3$$

Warning: $\frac{f'}{g'} \neq \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

L's Rule is only for $0/0$ & $\pm\infty/\infty$

$$\lim_{x\to\infty} \left(\ln(x^2+1) - \sqrt[3]{x}\right) = \lim_{x\to\infty} \left(\ln(x^2+1) - x^{1/3}\right)$$

$$= \cancel{\lim_{x\to\infty} x^{1/3}\left(\frac{\ln(x^2+1}{x^{1/3}} - 1\right)}$$

Plug in $x = H$

$$= \lim_{x\to\infty} \ln(x^2+1)\cdot\left(1 - \frac{x^{1/3}}{\ln(x^2+1)}\right) \quad \begin{matrix}\nearrow \infty \\ \searrow \infty\end{matrix} = \lim_{x\to\infty} \underbrace{\ln(H^2+1)}_{\text{+infinite}} \cdot \underbrace{\left(1 - \frac{H^{1/3}}{\ln(H^2+1)}\right)}_{\text{use L's Rule}}$$

$\Rightarrow$ To handle $\lim_{x\to ...} (f(x) - g(x)) =$ " $\infty - \infty$ ",
guess which one of f, g grows more slowly +
factor it out.

$$\lim_{x\to\infty} \frac{x^{1/3}}{\ln(x^2+1)} = \lim_{x\to\infty} \frac{(x^{1/3})'}{(\ln(x^2+1))'} = \lim_{x\to\infty} \frac{1/(3x^{2/3})}{2x/(x^2+1)} =$$

$$\lim_{x\to\infty} \frac{x^2+1}{2x\cdot 3x^{2/3}} = \lim_{x\to\infty} \frac{x^2+1}{6x^{5/3}} = \lim_{x\to\infty}\left(\underbrace{\frac{x^{1/3}}{6}}_{\infty} + \underbrace{\frac{1}{6x^{5/3}}}_{0}\right) = \boxed{\infty}$$

infinite

$$\underbrace{\ln(H^2+1)}_{\text{infinite}} \cdot \left(1 - \frac{H^{1/3}}{\ln(H^2+1)}\right) = \text{+(infinite)}(-\text{infinite}) = -\text{infinite} = \boxed{-\infty}$$