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Notes

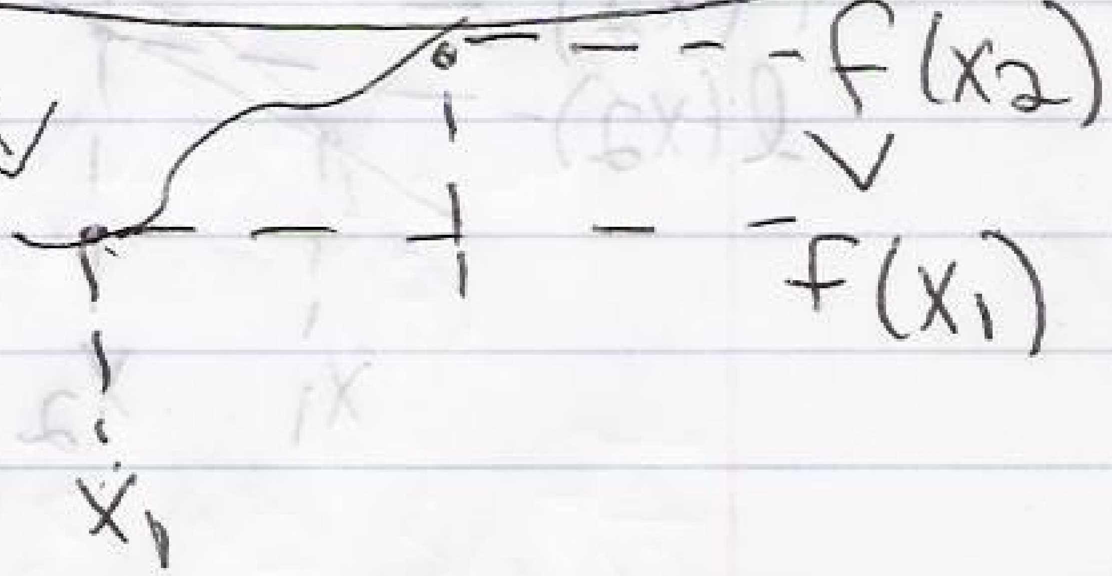
Today: $\nearrow$, $\searrow$, $\cup$, $\cap$ (3.7)

A function f is increasing ($\nearrow$) on
decreasing ($\searrow$)
concave up (CU or $\cup$)
concave down (CD or $\cap$)

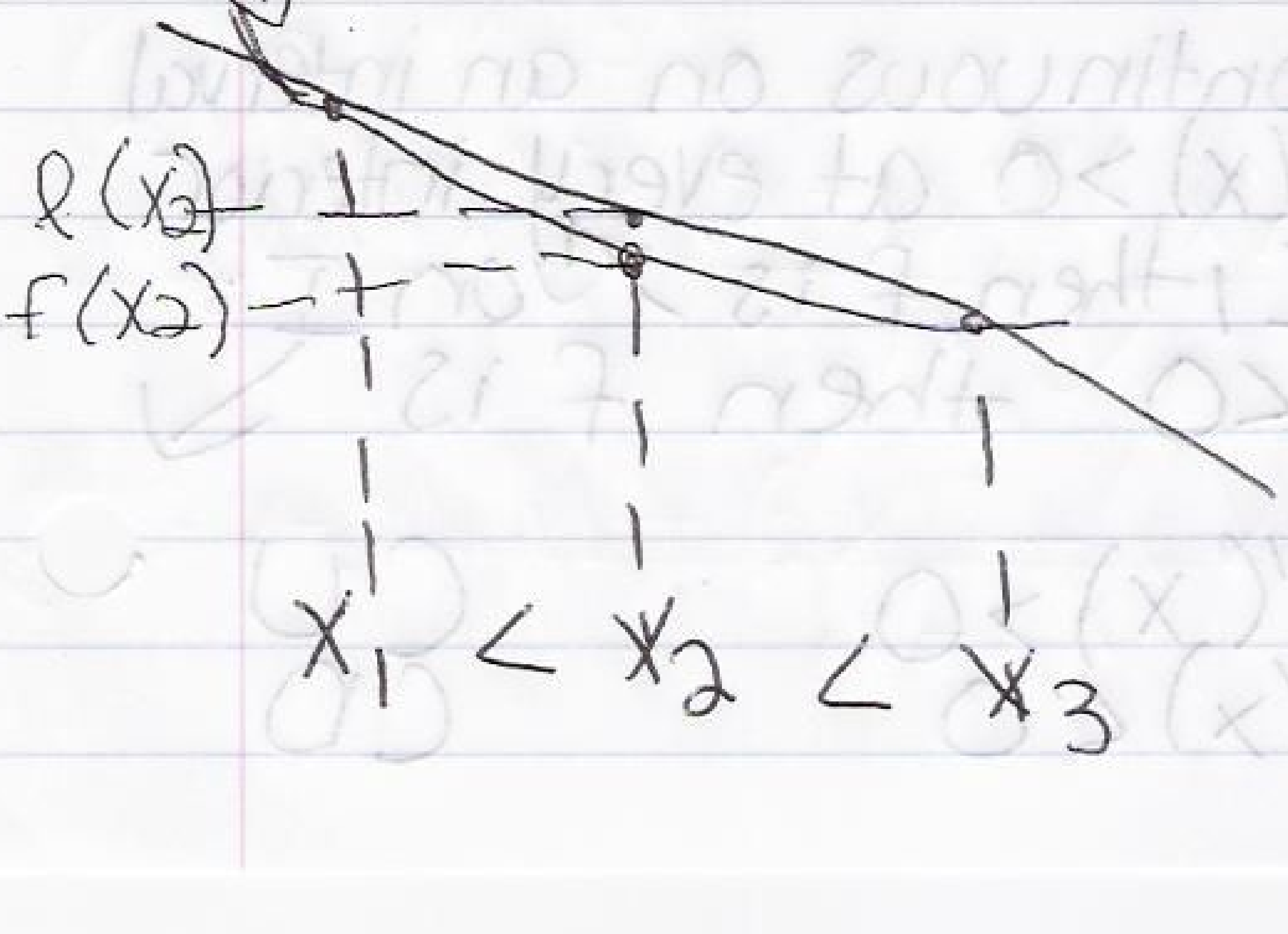
an interval I if for all

$x_1 < x_2$ in I,
$x_1 < x_2$ in I,
$x_1 < x_2 < x_3$ in I,
$x_1 < x_2 < x_3$ in I

$f(x_1) < f(x_2)$
$f(x_1) > f(x_2)$
$l(x_2) > f(x_2)$
$l(x_2) < f(x_2)$

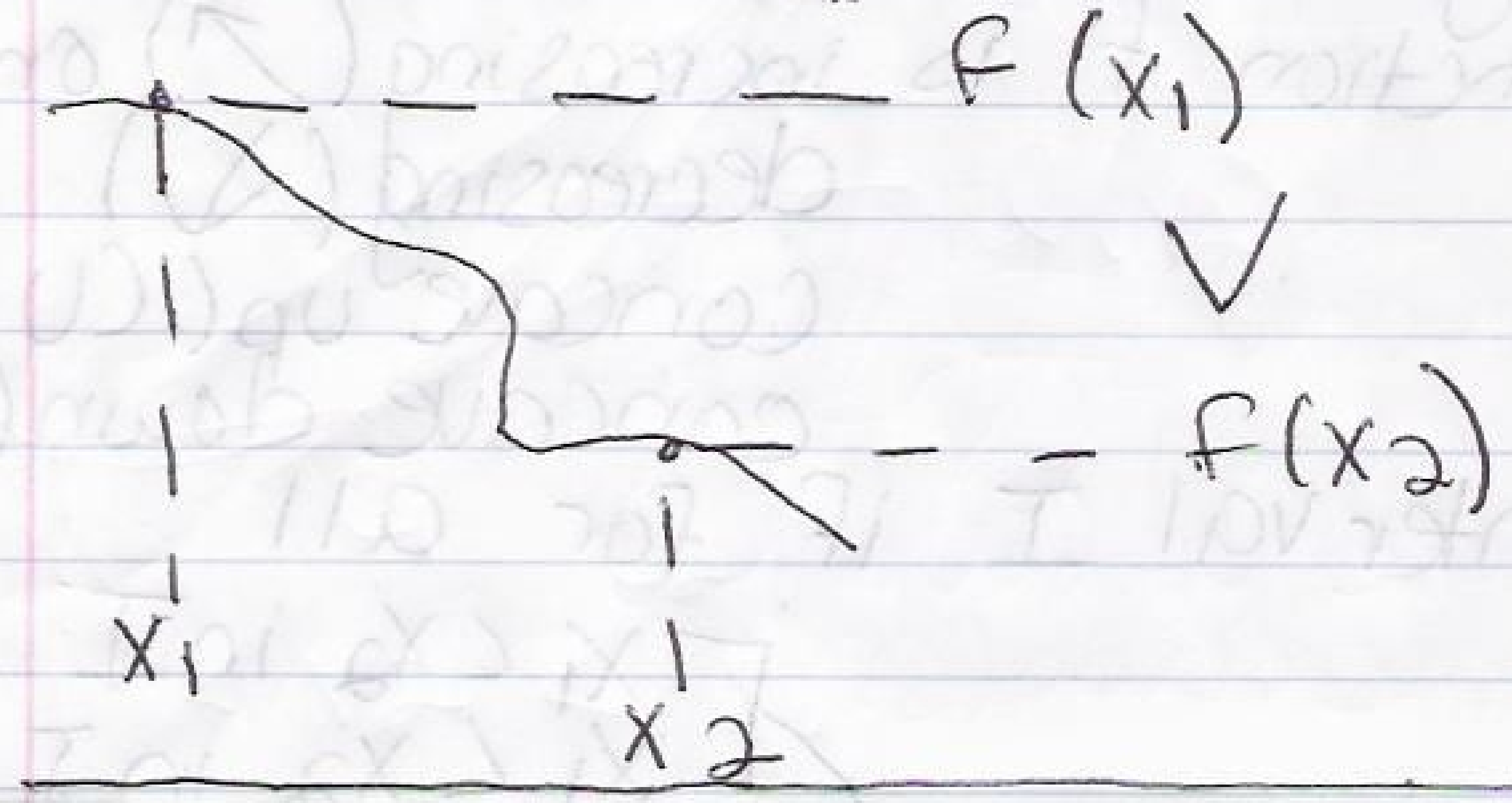


Curve: $y = f(x)$
Secant from x_1 to x_3: $y = l(x)$

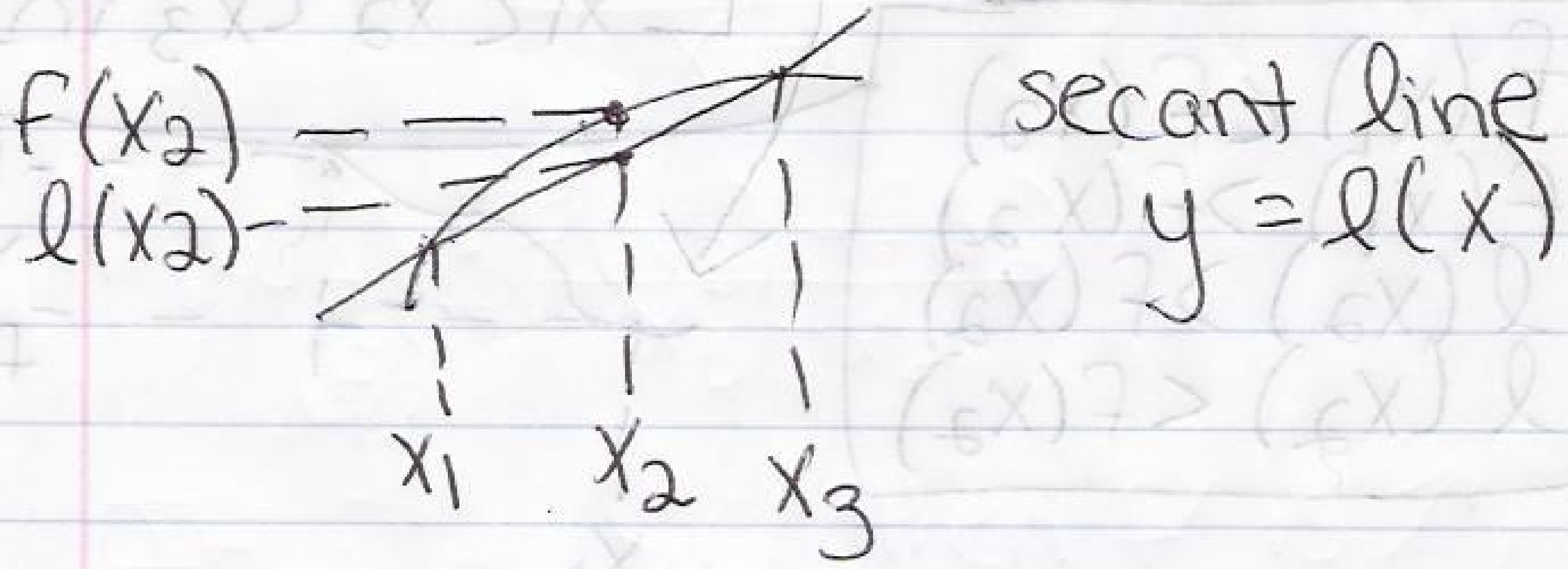


$$l(x) = \frac{f(x_3) - f(x_1)}{x_3 - x_1}(x - x_1) + f(x_1)$$

Picture of $\searrow$



Picture of CD or $\frown$

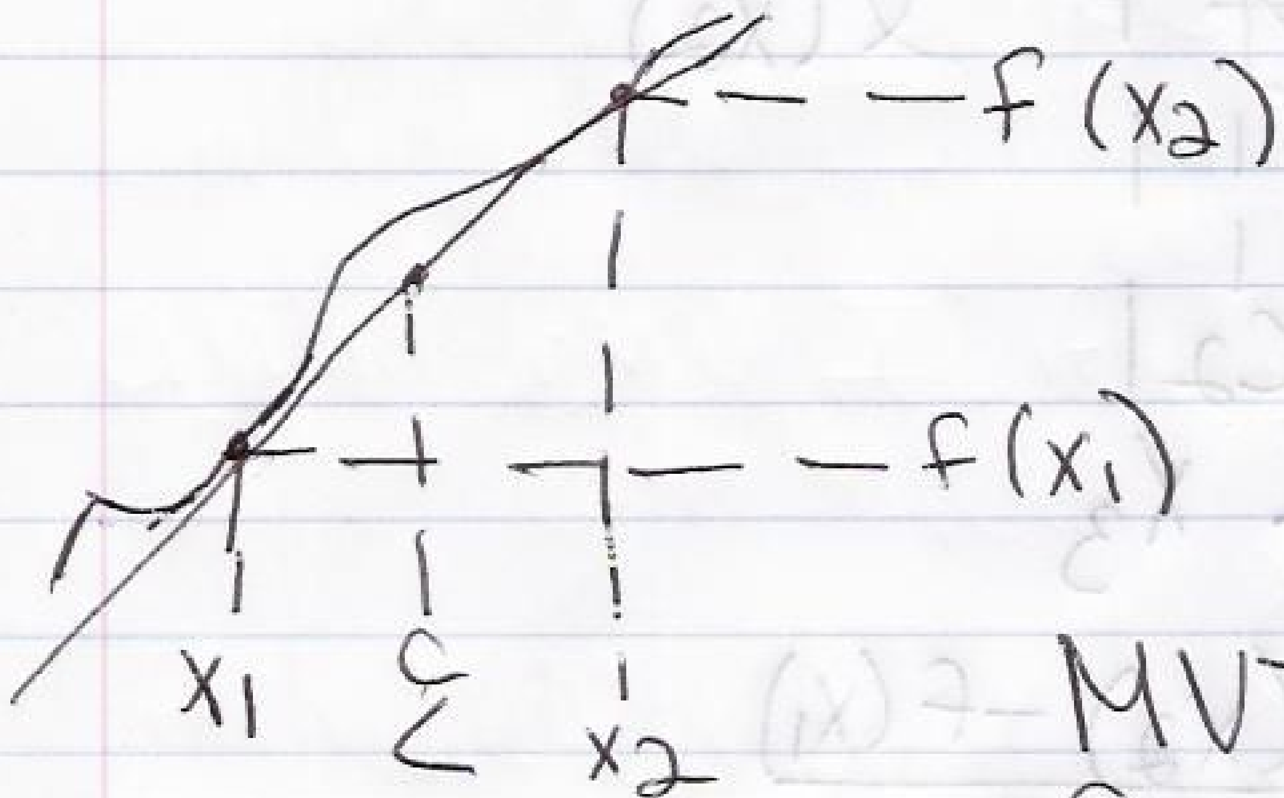


Terminology:
"Chord" = "secant line segment"

If f' is continuous on an interval I and $f'(x) > 0$ at every interior point of I, then f is $\nearrow$ on I. If $f'(x) < 0$ then f is $\searrow$ on I.

$f''(x) > 0$ CU
$f''(x) < 0$ CD

Why? The trick is to use the Mean Value Theorem.



MVT says there some c with

$x_1 < c < x_2$ &

$$\underbrace{f'(c)}_{\text{tangent slope}} = \underbrace{\frac{f(x_2)-f(x_1)}{x_2-x_1}}_{\text{secant slope}} \Rightarrow$$

$$= f(x_2)-f(x_1) = f'(c)\underbrace{(x_2-x_1)}_{\text{positive}}$$

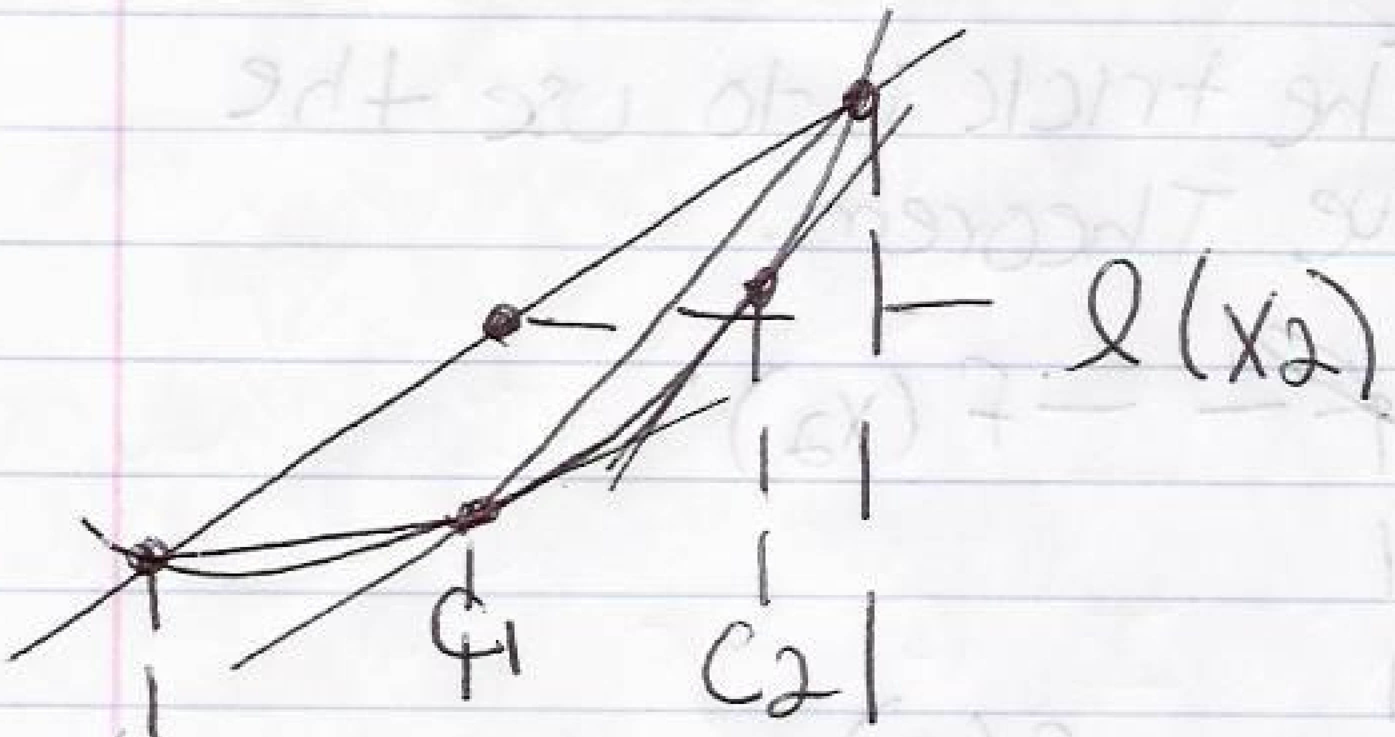
f is $\nearrow$ ~~#1~~

$f(x_1) < f(x_2)$

So, $f'(c) > 0 \Rightarrow f(x_2) - f(x_1) > 0$.

$f'(c) < 0 \Rightarrow f(x_2) - f(x_1) < 0 \Rightarrow$

$= f(x_1) > f(x_2)$ = f is $\searrow$



$x_1 < x_2 < x_3$

$$f'(c_1) = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$f'(c_2) = \frac{f(x_3)-f(x_2)}{x_3-x_2}$$

$(f')'$

If f'' is >0, then f' is $\nearrow$, so $f'(c_1) < f'(c_2)$!

Consider $\ell(x_2) - f(x_2) = \frac{f(x_3)-f(x_1)}{x_3-x_1}(x_2-x_1) - f(x_2)$

$$= \frac{(f(x_3)-f(x_1))(x_2-x_1) - f(x_2)(x_3-x_1)}{x_3-x_1}$$

$$= \frac{(f(x_3)-f(x_2)+f(x_2)-f(x_1))(x_2-x_1) - f(x_2)(x_3-x_1)}{x_3-x_1}$$

= . . . is > 0

$f(x)=x^4-6x^2 = x^2(x^2-6) = x^2(x-\sqrt{6})(x+\sqrt{6})$

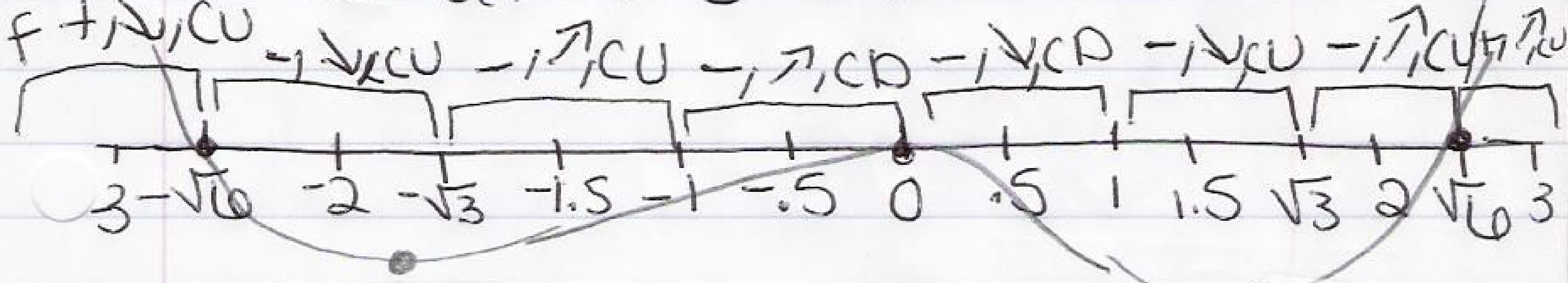
$f'(x) = 4x^3-12x = 4x(x^2-3) = 4x(x-\sqrt{3})(x+\sqrt{3})$

$f''(x) = 12x^2-12 = 12(x^2-1) = 12(x-1)(x+1)$

is 0 at $x = 0 \pm \sqrt{6}$

is 0 at $x = 0 \pm \sqrt{3}$

is 0 at $x = 0 \pm 1$



f   0 . . – – – – 5 – – 0 – – –5 – – – – 0 + + 27

f′   – – – 8 – 0 + + 8 + + 0 – – 8 – – 0 + + 8 + + +

f″   + + + 36 + + + 0 – – 9 – 0 – – 9 – – – 0 + + 36 + +