11/4/10

$f(x) = x^5 - 10x^3$
$g(x) = 1/x$
$h(x) = x^4$
$u(x) = x^{2/3}$

Find the maximal intervals on which f(x) is CU → $(-\sqrt{3}, 0]$ and $[\sqrt{3}, \infty)$

"maximal" means impossible to extend further.

$f'(x) = 5x^4 - 30x^2$
$f''(x) = 20x^3 - 60x = 20x(x^2-3) = 20x(x-\sqrt{3})(x+\sqrt{3})$ is $=0$ at $x = 0, \pm\sqrt{3}$

x	-2	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$	2
$f''(x)$	-40	0	40	0	-40	0	40

$- - - - - 0 + + + + 0 - - - - + + + +$

f(x)	CD	CU	CD	CU
	$(-\infty, -\sqrt{3}]$	$[-\sqrt{3}, 0]$	$[0, \sqrt{3}]$	$[\sqrt{3}, \infty)$

find the maximal intervals on which f is ↘ (decreasing)

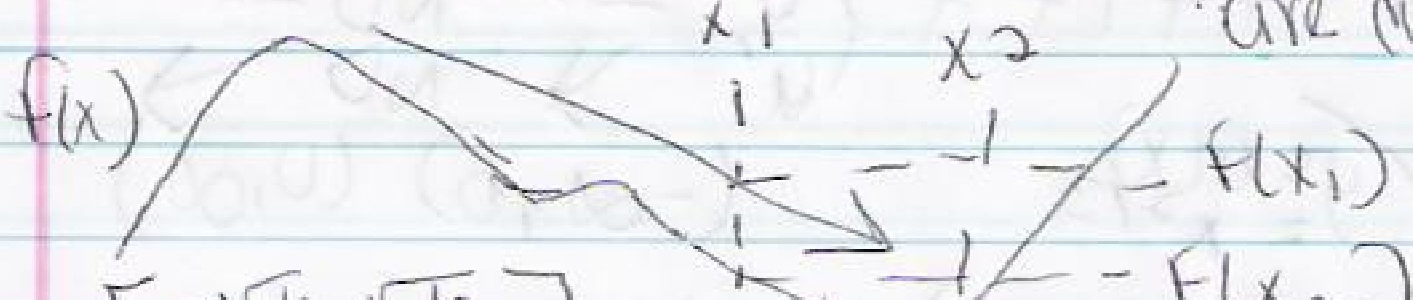
$f'(x) = 5x^4 - 30x^2 = 5x^2(x^2-6) = 5x^2(x-\sqrt{6})(x+\sqrt{6})$ is $=0$ at $x = 0, \pm\sqrt{6}$

$x^2 - 6$
$= x^2 - \sqrt{6}^2$
$= (x-\sqrt{6})(x+\sqrt{6})$

$a^2 - b^2 = (a-b)(a+b)$

x	-3	$-\sqrt{6}$	-1	0	1	$\sqrt{6}$	3
$f'(x)$	135	0	-25	0	-25	0	135

$+ + 0 - - 0 - - 0 + + + +$



$[-\sqrt{6}, \sqrt{6}]$

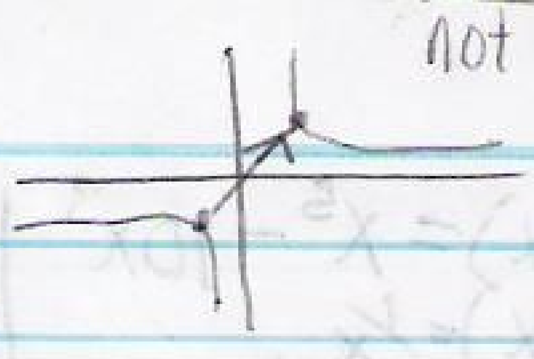
You can combine intervals on which f is ↘ if the intervals are connected (likewise for ↗). $[-\sqrt{6}, 0]$ & $[0, \sqrt{6}]$ are connected at 0.

find the maximal intervals on which $g(x)$ $\searrow$ $\boxed{(-\infty,0) \text{ and } (0,\infty)}$

$g'(x) = -1/x^2$ is ND at $x=0$ & is never $=0$

not connected

x	-1	0	1
$g'(x)$	-1	ND	-1
	– – – – – –	ND	– – – – –
$g(x)$	$\searrow$	ND	$\searrow$



find the maximal interval on which $h(x)$ is CU: $\boxed{(-\infty,\infty)}$

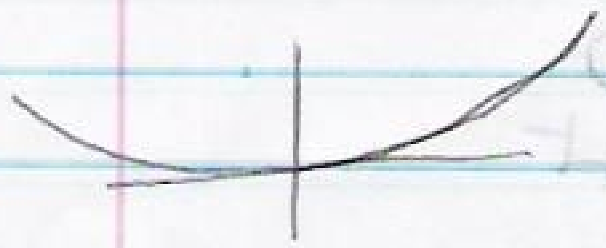
$h'(x) = 4x^3$ $h'' = 12x^2$ is $=0$ at $x=0$

x	-1	0	1
$h''(x)$	12		12
	+ + + + + +	0	+ + + + +
$h(x)$	CU		CU
	$(-\infty, 0]$		$[0,\infty)$

is $h(x)$ CU on $(-\infty, \infty)$?

→ yes: ~~[illegible]~~ $h'' > 0$ on $(0,\infty)$ & $h'' = (h')'$ & h' is cts (everywhere), h' is $\nearrow$ on $[0,\infty)$

Similarly h' is $\nearrow$ on $(-\infty,0]$ so we can combine $(-\infty,0]$ & $[0,\infty)$: h' is $\nearrow$ on $(-\infty,\infty)$, so h is CU on $(-\infty,\infty)$.



find the maximal intervals on which $u(x)$ is CD.

$(-\infty,0]$ and $[0,\infty)$

$$u'(x) = \frac{2}{3}x^{2/3-1} = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

$$\rightarrow u''(x) = \frac{2}{3}\cdot\left(-\frac{1}{3}\right)x^{-\frac{1}{3}-1} = -\frac{2}{9}x^{-\frac{4}{3}} = \frac{-2}{9\sqrt[3]{x^4}} = \frac{-2}{9(\sqrt[3]{x})^4}$$

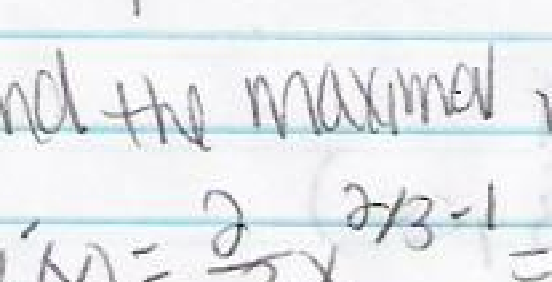
is ND at $x=0$ and is never $=0$

x	-1	0	1
u''	-2/9	ND	-2/9
	– – – – – –	ND	– – – – –
u	CD		CD
	$(-\infty, 0]$		$[0,\infty)$
	+ + + + + +	0	+ + + + + +

Is u CD on $(-\infty,\infty)$? NO:

~~x -1 0 1~~
~~u' -2/3 ND 2/3~~
~~– – – ND + + + + +~~

x		0	
u''	– – –	ND	– – – –
u'	$\searrow$	ND	$\searrow$
	$(-\infty,0)$		$(0,\infty)$



$y = u(x) = x^{2/3}$

CD on $(-\infty,0]$ CD on $[0,\infty)$