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Today: Local maxima/minima

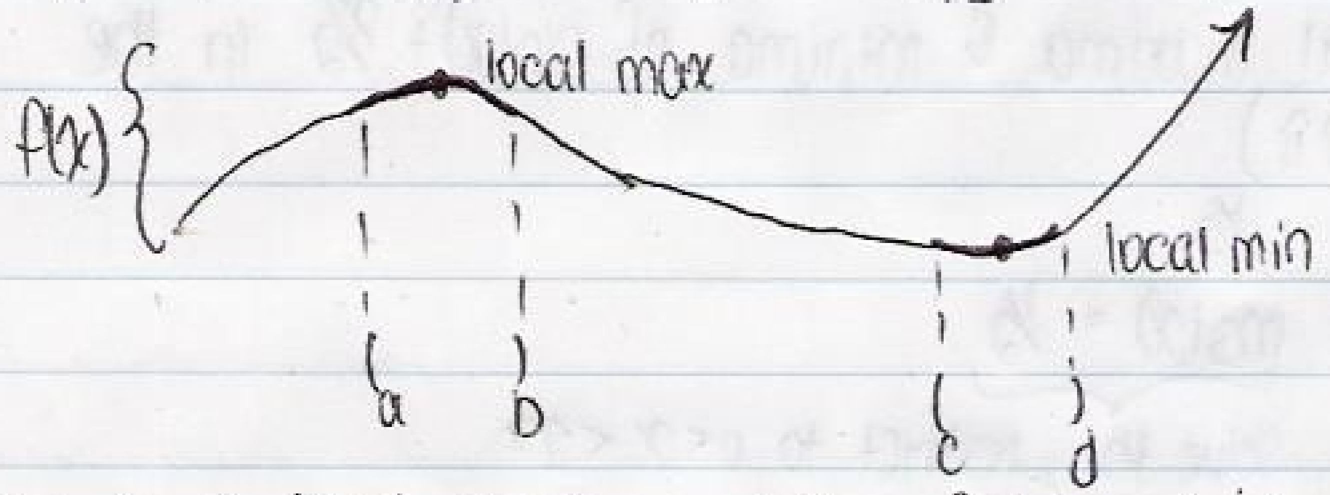
$f(c)$ is a local maximum value of $f(x)$ if, for some open interval (a,b) with $a<c<b$, $f(c)$ is the maximum value of $f(x)$ over (a,b)

$f(x) = x^3 - x = x(x^2-1)$

$f'(x) = 3x^2 - 1$

$f'(x) = 0 \iff 3x^2 = 1 \iff x = \pm 1/\sqrt{3}$

$$\begin{array}{lccccc} & \{+\,+\,+\,+ & 0 & -\,-\,-\,-\,-\,- & 0 & +\,+\,+\,+ \\ f'(x) & 2 & 0 & -1 & 0 & 2 \\ x & -1 & -1/\sqrt{3} & 0 & 1/\sqrt{3} & 1 \end{array}$$



There is a local maximum value $f\left(\frac{-1}{\sqrt{3}}\right) = \frac{-1}{\sqrt{3}}\left(\frac{1}{3}-1\right) = \frac{2}{3\sqrt{3}}$ at $x = -1/\sqrt{3}$

There is a local minimum value $f(1/\sqrt{3}) = 1/\sqrt{3}(1/3 - 1) = \frac{-2}{3\sqrt{3}}$ at $x = 1/\sqrt{3}$

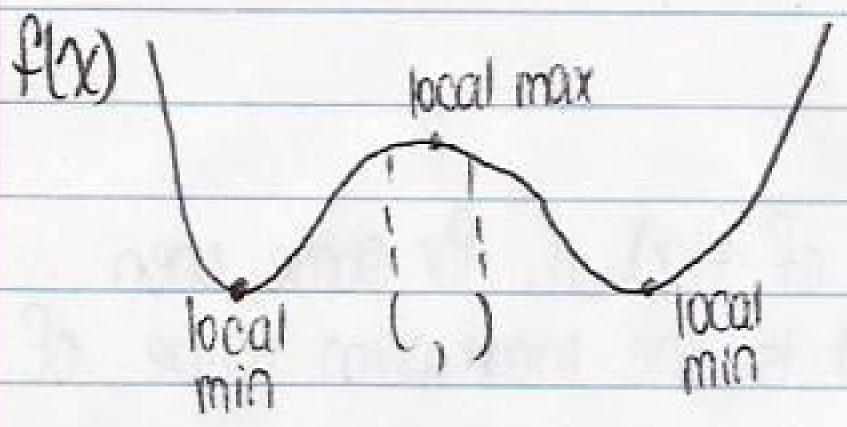
If f is $\uparrow$ on $[a,c]$ and $\downarrow$ on $[c,b]$ then f has a local max. value $f(c)$ at $x=c$

$f(x) = x^4 - x^2$

$f'(x) = 4x^3 - 2x = 2x(2x^2-1) = 2x(\sqrt{2}x+1)(\sqrt{2}x-1)$

$f'(x) = 0 \iff x = 0, \pm 1/\sqrt{2}$

$$\begin{array}{lccccc} & \{-\,-\,-\,- & 0\,+\,+\,+ & 0\,-\,-\,- & 0\,+\,+\,+ & \\ f'(x) & -2 & 0 & 0 & 0 & \\ x & -1 & -1/\sqrt{2} & 0 & 1/\sqrt{2} & 1 \end{array}$$



<u>local max. values</u>

$f(0) = 0^4 - 0^2 = 0$

<u>local min. values</u>

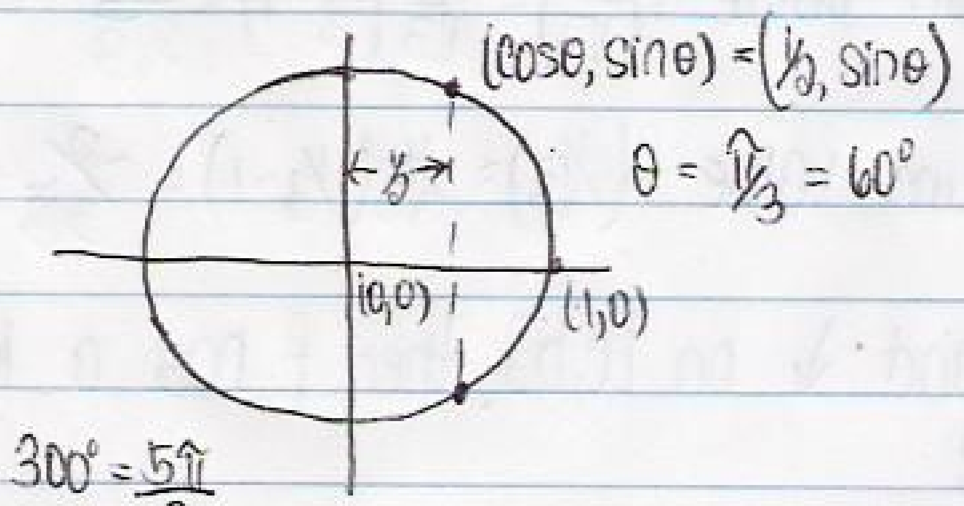
$f(-1/\sqrt{2}) = (-1/\sqrt{2})^4 - (-1/\sqrt{2})^2 = -1/4$
$f(1/\sqrt{2}) = (1/\sqrt{2})^4 - (1/\sqrt{2})^2 = -1/4$ } Also min. value of f over $(-\infty, \infty)$

Find the local maxima & minima of $\sin(x) + x/2$ in the interval $(0, 2\pi)$.

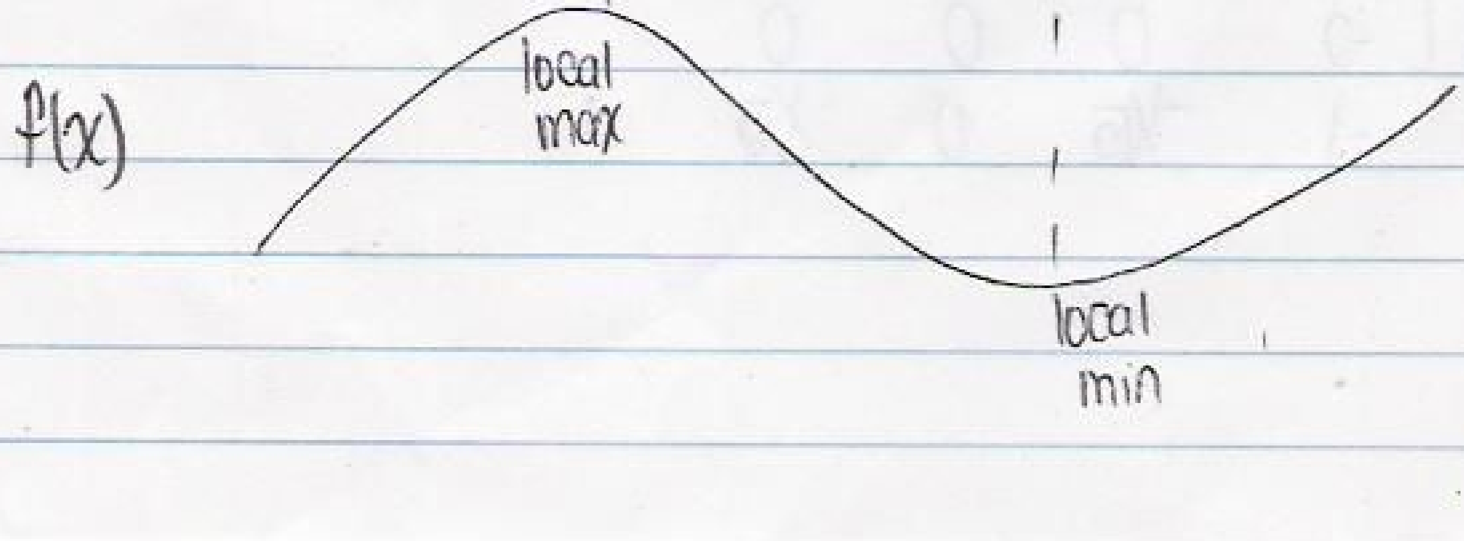
$f'(x) = \cos(x) - 1/2$

$f'(x) = 0 \iff \underbrace{\cos(x) = 1/2}$

Solve this; restrict to $0 < x < 2\pi$



x	0	$\pi/3$	π	$5\pi/3$	2π
	$0°$	$60°$	$180°$	$300°$	$360°$
$f'(x)$	$1/2$	0	$-3/2$	0	$1/2$
	+ + + + +	0	− − − −	0	+ + + + +



local max. value $= f(\pi/3) = \sqrt{3}/2 + \pi/6$

local min. value $= -\sqrt{3}/2 + \frac{5\pi}{6}$

$f(x) = x^3 - x^5 \qquad f'(x) = 3x^2 - 5x^4 = x^2(3-5x^2)$

$= x^2(\sqrt{3}-\sqrt{5}x)(\sqrt{3}+\sqrt{5}x)$

$x = 0, \pm\sqrt{3/5}$

x	-1	$-\sqrt{3/5}$	$-\sqrt{1/2}$	0	$\sqrt{1/2}$	$\sqrt{3/5}$	1
$f'(x)$	-2	0	$1/4$	0	$1/4$	0	-2

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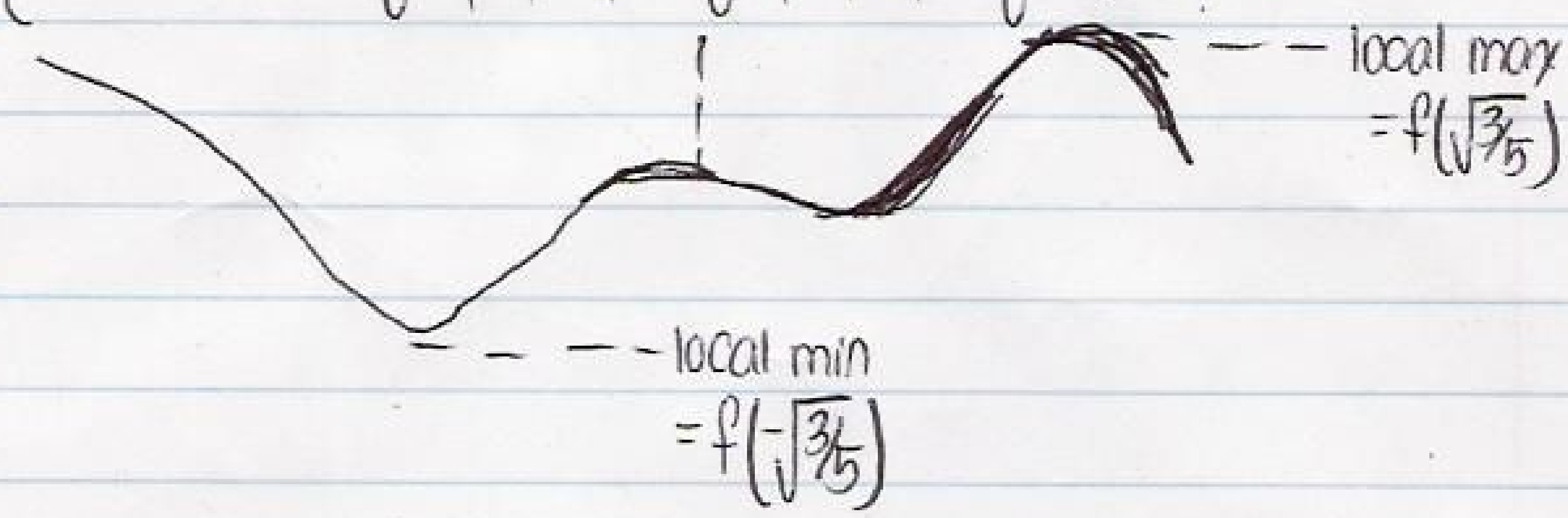

$x^3 - x^5$

has local max. value $(\sqrt{3/5})^3 - (\sqrt{3/5})^5$ at $x = \sqrt{3/5}$

$(-\sqrt{3/5})^3 - (-\sqrt{3/5})^5$ at $x = -\sqrt{3/5}$

$f(x) = x + 1/x \qquad f'(x) = 1 - 1/x^2 \leftarrow$ undefined at $x=0$

$0 = f'(x) \longleftrightarrow 1 = 1/x^2 \longleftrightarrow x^2 = 1, \; x = \pm 1$

x	-2	-1	$-1/2$	0	$1/2$	1	2
$f'(x)$	$3/4$	0	-3	UD	-3	0	$3/4$

$+ + + \; 0 \; - - - - \; \text{UD} \; - - - \; 0 \; + + + +$