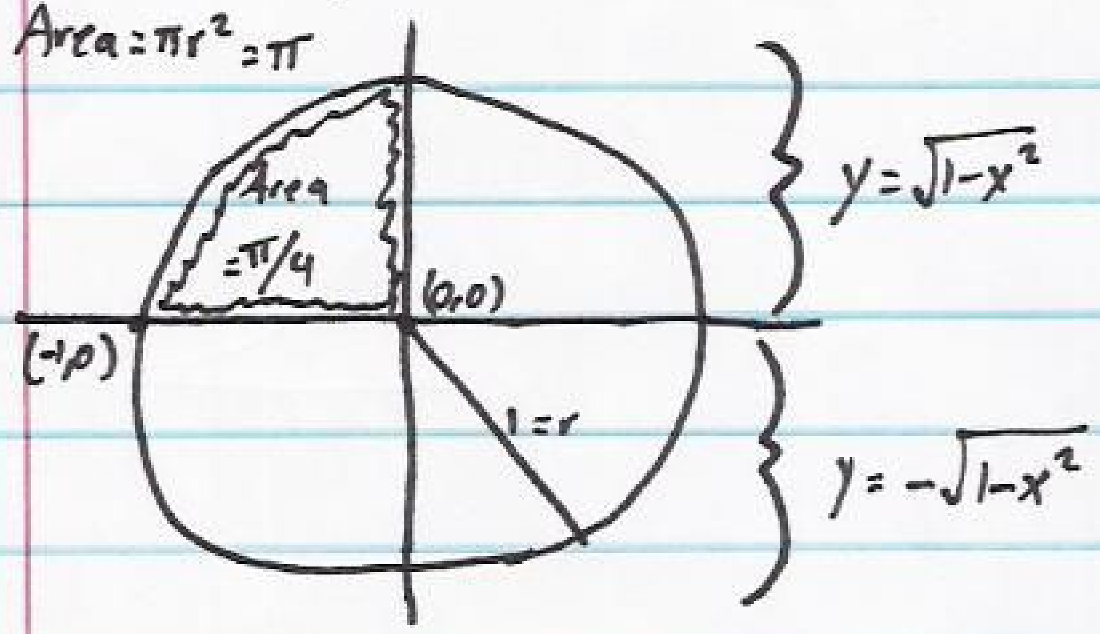
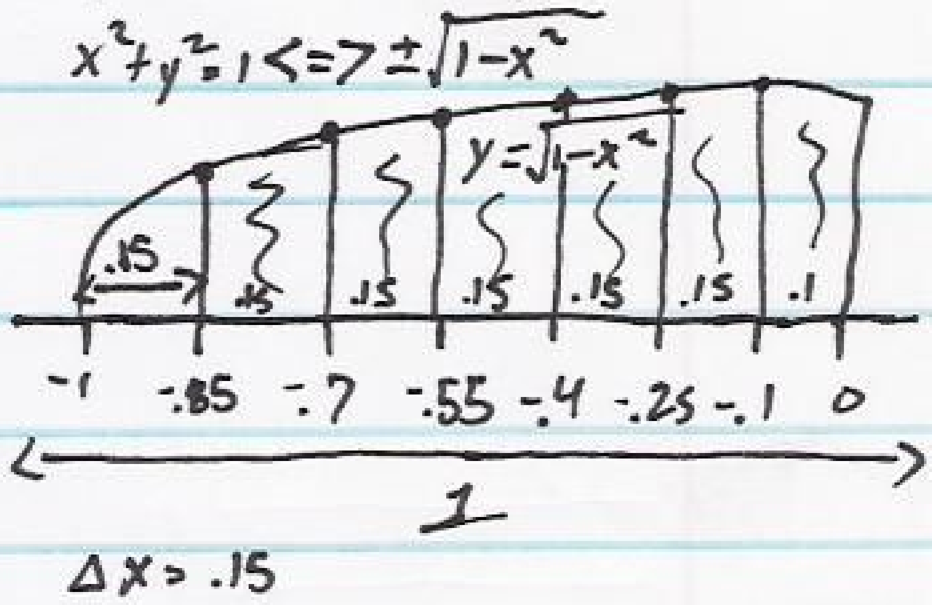
Starting with Section 4.1 Nov 10, 2010

- Idea: approximate area of curved shape with rectangles.

$x^2+y^2=1$





$x_0=-1$ $x_1=-.85$ $x_2=-.7$ $x_3=-.55$ $x_4=-.4$ $x_5=-.25$ $x_6=-.1$ $x_7=0$

$[x_0,x_1]$ $[x_1,x_2]$ $[x_2,x_3]$ $[x_3,x_4]$ $[x_4,x_5]$ $[x_5,x_6]$ $[x_6,x_7]$

$\sqrt{1-(-1)^2}=0=f(-1)$ $\sqrt{1-(-.85)^2}=f(-.85)$ $\sqrt{1-(-.7)^2}=f(-.7)$ $y=\sqrt{1-x^2}=f(x)$

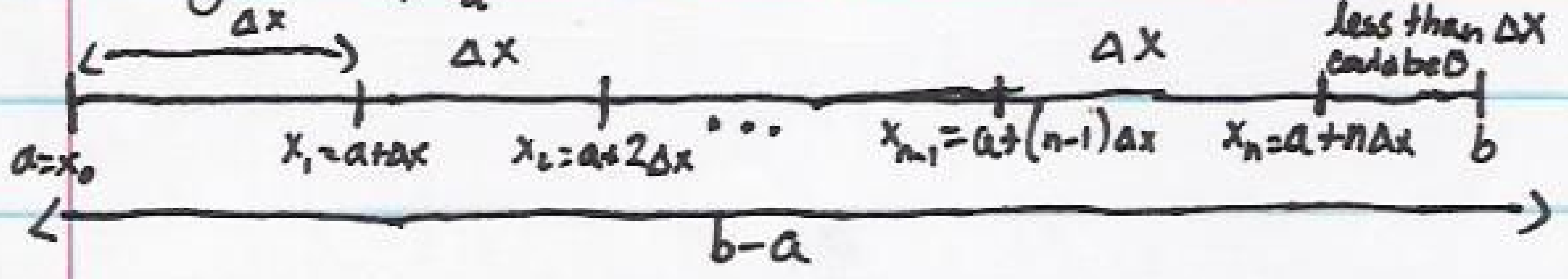
- Estimate for area ($\pi/4$):

$f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta x + f(x_6)\underbrace{(0-x_6)}_{0-(-.1)=.1}$

called Riemann Sum. Book's notation: $\sum_{-1}^{0} f(x)\Delta x$

n is greatest integer

☆ In general, $\sum_a^b f(x)\Delta x = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_{n-1})\Delta x + f(x_n)(b-x_n)$ where $a+n\Delta x \le b$



$\boxed{\text{Extreme Value Theorem}}$ →

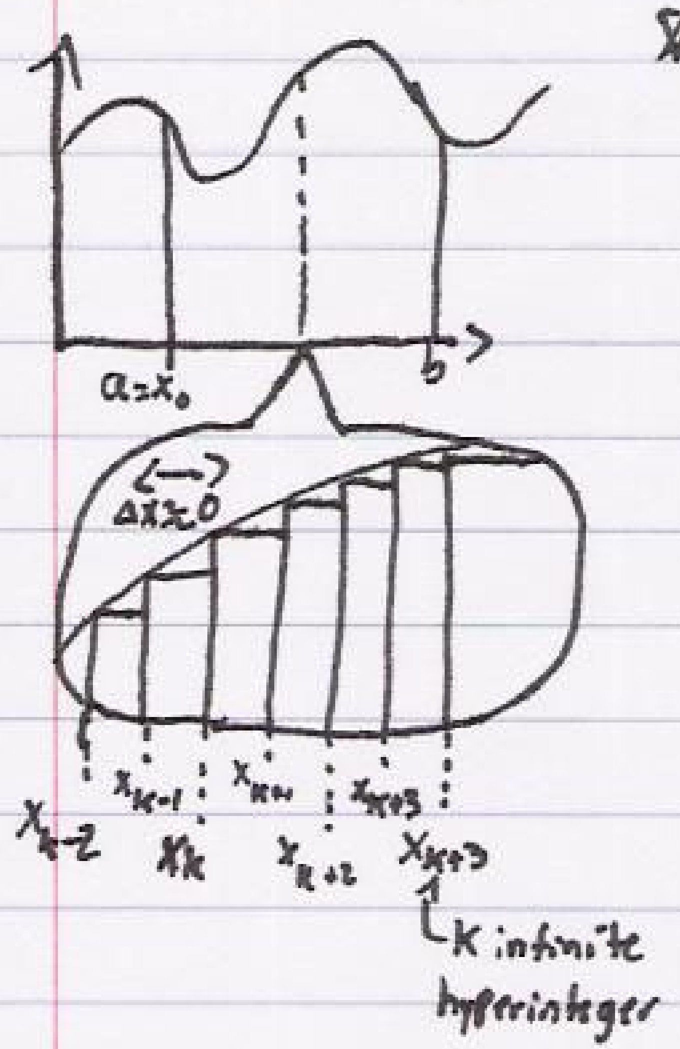
- If f is continuous on $[a,b]$, then f has a min. value m & a max. value M, over $[a,b]$.

f cts. on $[a,b] \Rightarrow m(b-a) \le \sum_a^b f(x)\Delta x \le M(b-a)$

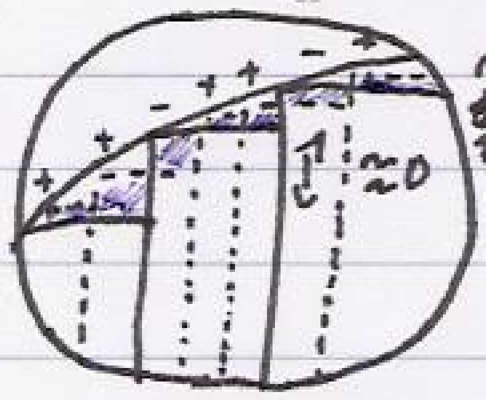
$m=\min(f)$ over $[a,b]$ $M=\max(f)$ over $[a,b]$

- By Transfer, since $m(b-a) \le \sum_a^b f(x)\Delta x \le M(b-a)$ for all real $\Delta x>0$, it's also true for all hyperreal $\Delta>0$. So, $\sum_a^b f(x)\Delta x$ is <u>finite</u>, even when $\Delta x \approx 0$

This assume f(x) is cts. on [a, b]



☆When $0<\Delta x \approx 0$ & f is cts. on [a, b], then $\sum_a^b f(x)\Delta x$ is a finite hyperreal & if you a different width, say $0<\Delta x \approx 0$, that is still infinitesimal, then $\sum_a^b f(x)\,s \approx \sum_a^b f(x)\Delta x$



☆Definition: Assuming f is cts. & $dx \approx 0$. $\int_a^b f(x)\,dx = st\left(\sum_a^b f(x)\,dx\right)$

round of f to nearest real.

$\int_{-1}^{0} \sqrt{1-x^2}\,dx = \pi/4$