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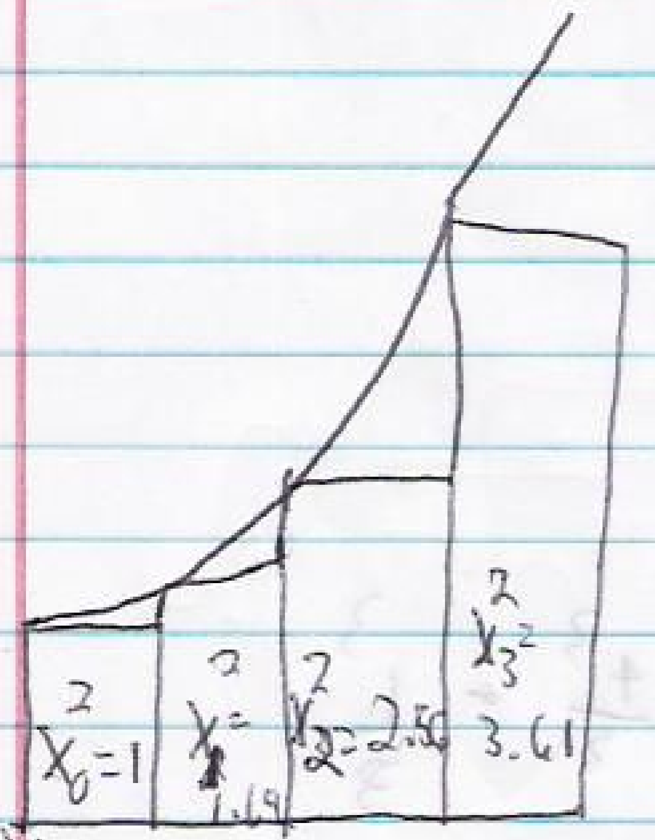
Integration, continued

$\sum_{1}^{2} x^2 \Delta x = ?$ if $\Delta x = 0.3$

$\sum_{1}^{2} x^2 \Delta x = (1)^2(.3) + (1.6)^2(.3) = .3 + .12 =$

$\sum_{1}^{2} x^2 \Delta x =$

$\Delta x = .3$	$\Delta x = .3$	$\Delta x = .3$	$\Delta x = .1$	
x_0	x_1	x_2	x_3	x_4
1	1.3	1.6	1.9	2
	$1+\Delta x$	$1+2\Delta x$	$1+3\Delta x$	



$x_b =$ (left end point) $+ b\Delta x$
except maybe last x_i

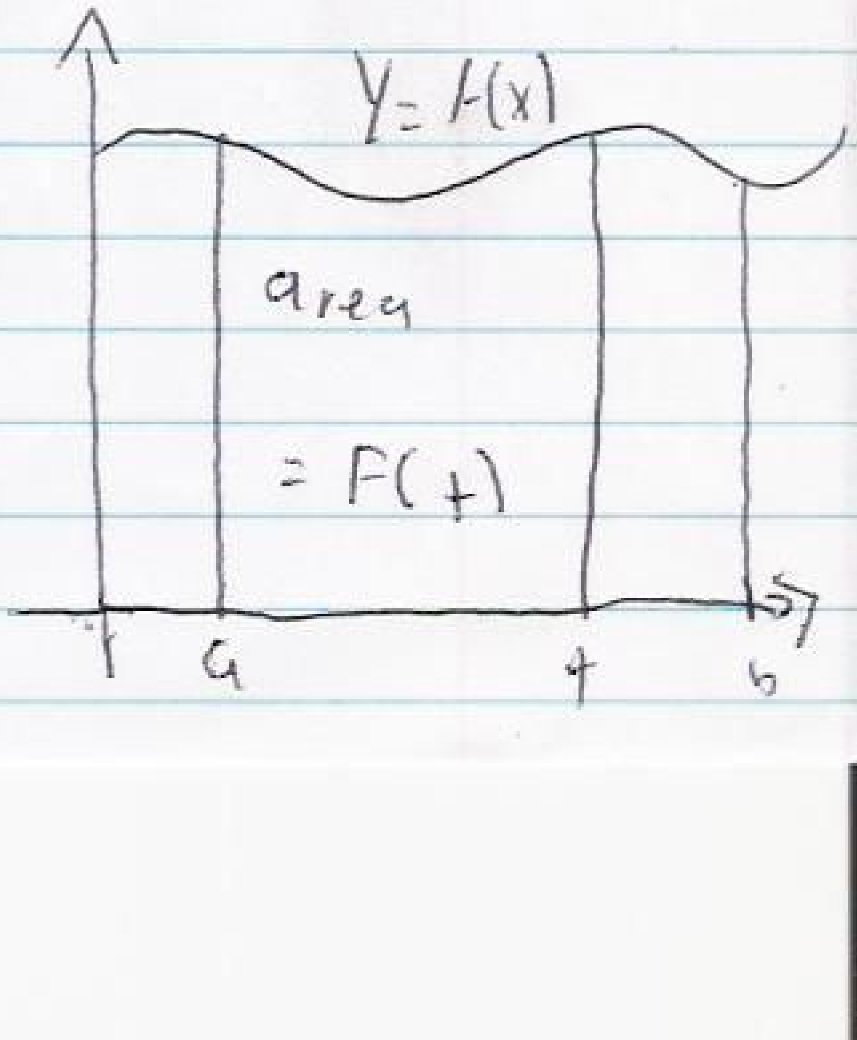
$\sum_{1}^{2} x^2 \Delta x = 1(.3) + (1.69(.3)) + 2.56(.3) + 3.61(.1)$

$= 1.936$

Using derivatives to find integrals
Assume f is continuous on $[a,b]$

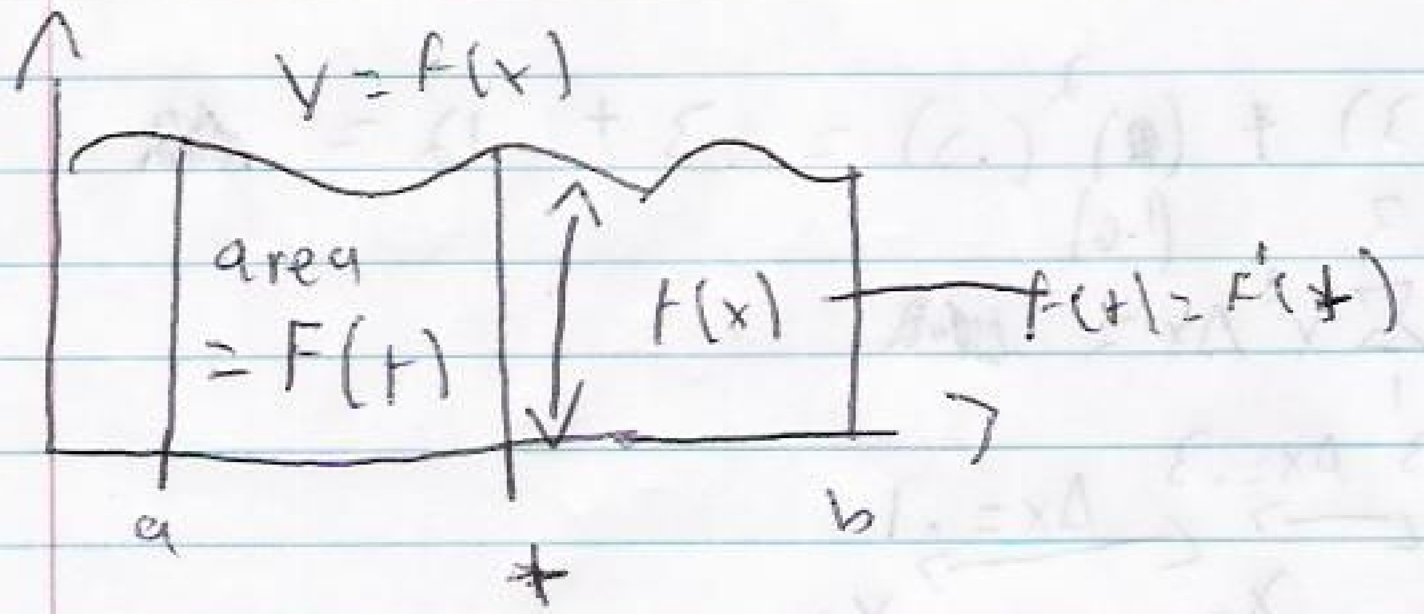
For all t in $[a,b]$,

$$F(t) = \int_a^t f(x)\,dx$$

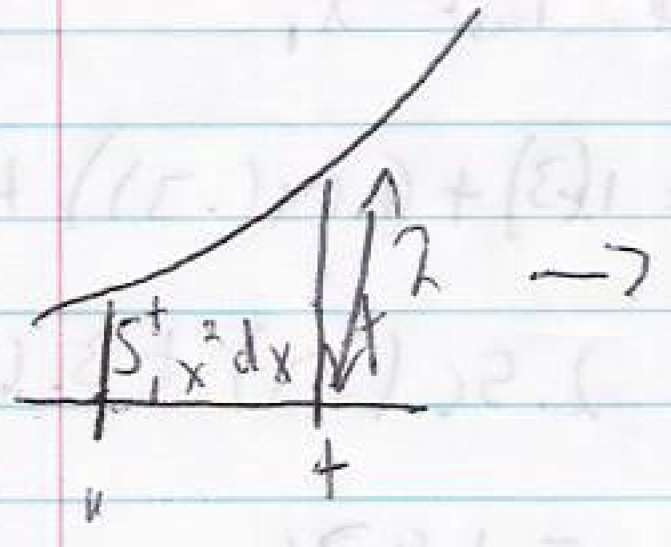


1st Fundamental Theorem of Calculus

$$F'(t) = \frac{d\left(\int_a^t f(x)\,dx\right)}{dt} = \left(\int_a^t f(x)\,dx\right)' = f(t)$$



Eg. $\left(\int_1^t x^2\,dx\right)' = t^2$



This implies

$$\int_1^t x^2\,dx = \frac{t^3}{3} - \frac{1}{3}$$

because $\left(\frac{t^3}{3}\right)' = t^2$

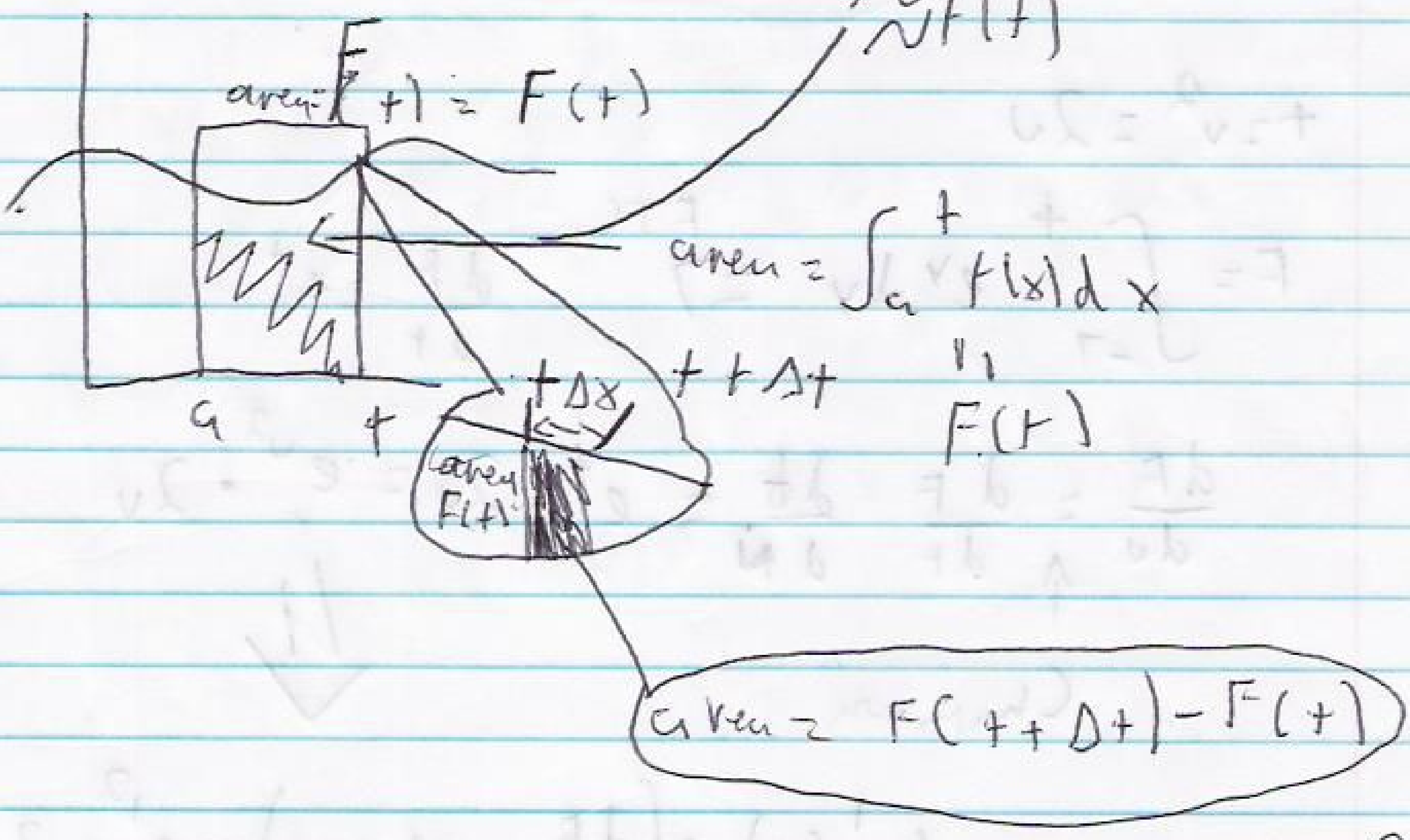
why does this work?

$$F'(t) = st\left(\frac{F(t+\Delta t)-F(t)}{\Delta t}\right)$$

where $0 \neq \Delta t \approx 0$

$F(t+\Delta t) - F(t)$

$$\int_a^{t+\Delta t} f(x)\,dx - \int_a^{t} f(x)\,dx$$



width $= \Delta t$

$\Downarrow$

$$\frac{F(t+\Delta t) - F(t)}{\Delta t} = \frac{\text{area}}{\text{width}} = \text{avg. height}$$

$\approx f(t)$ because F is cts.

$F'(t) = st\left(F(t+\Delta t) - F(t) \right)$

FTC_1: If f is cts. on $[a,b]$ & $a < t < b$,

and $F(x) = \int_a^t f(x)\,dx$

then $F'(t) = f(t)$

E.g. $F(v) = \int_{-7}^{v^2} e^x\,dx$

$F'(1) =$

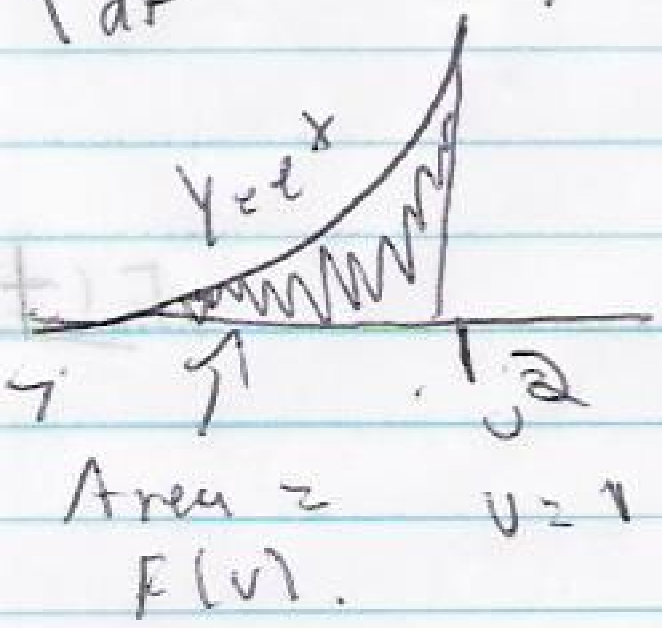
$t = v^2 = 2v$

$F = \int_{-7}^{t} e^y\,dy \overset{FTC_1}{\Longrightarrow} \frac{dF}{dt} = e^t$

$\frac{dF}{dv} = \frac{dF}{dt}\frac{dt}{dv} = e^t \cdot 2v = e^{v^2} \cdot 2v$

Chain rule

$F'(1) = \left(\frac{dF}{dt} \text{ at } v = 1\right) = e^{1^2} \cdot 2 \cdot 1 = 2e$



FTC_2 : If f & G are CTS on $[a,b]$ and

$G'(t) = f(t)$ for all t in (a,b)

then $\displaystyle\int_a^b f(x)\,dx = G(b) - G(a)$

Proof using FTC_1 :

Define

$F(t) = \displaystyle\int_a^t f(x)\,d(x)$

Then $G'(t) = f(t) = F'(t)$
$\uparrow$
FTC_1

So, $G'(t) = F'(t)$
Using the mean value theorem, you can prove that ~~xxxx~~ Since

$G' = F'$, we have
$G(t) = F(t) + C$
for some constant C.

$$\int_a^b f(x)\,dx = \int_a^b f(x)\,dx - \underbrace{\int_a^a f(x)\,dx}_{\substack{\text{area} = 0 \\ \text{because width} = a - a = 0}}$$

$= F(b) - F(a)$
$= F(b) - F(a) + C - C$
$= (F(b) + C) - (F(a) + C)$
$= G(b) - G(a)$

$$\sum_{1}^{2} x^2 \Delta x = 1.936$$
when $\Delta x = 0.3$

$$\int_1^2 x^2\,dx = \frac{2^3}{3} - \frac{1^3}{3} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} = 2.3333\ldots$$

$$G'(t) = \left(\frac{t^3}{3}\right)' = \frac{3t^2}{3} = t^2 = f(t)$$

$$G(t) = \frac{t^3}{3} \qquad f(t) = t^2$$

Shorter notation for using FTC$_2$:

If $G'(x) = f(x)$, then $\int_a^b f(t)\,dt = \underbrace{G(t)\Big|_a^b}$ means $G(b) - G(a)$

(assuming f is cts. on $[a,b]$)