Recap:

If f is cts on $[a,b]$ and $a<t<b$, then

(FTC$_1$) $\left(\int_a^t f(x)\,dx\right)' = f(t)$

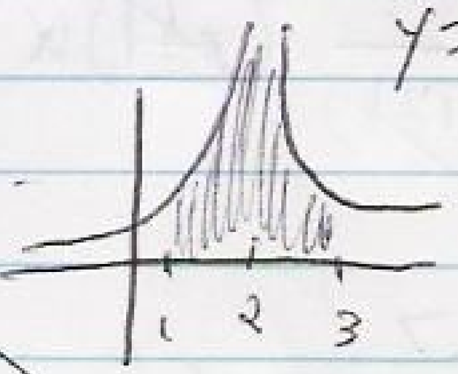
(FTC$_2$) $\int_a^b f(x)\,dx = G(b) - G(a)$

If $G'(x) = f(x)$

You really do need continuity for this to work:

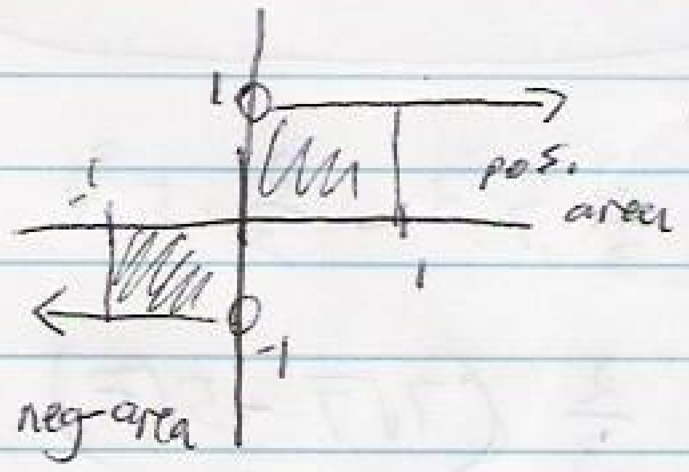
ex.1

$\int_1^3 \frac{1}{(x-2)^2}\,dx$ is not defined



ex.2

$\left(\int_{-1}^t \frac{|x|}{x}\,dx\right)'$ is undefined at $t=0$



$y = \frac{|x|}{x} = \begin{cases} 1 : x>0 \\ -1 : x<0 \end{cases}$

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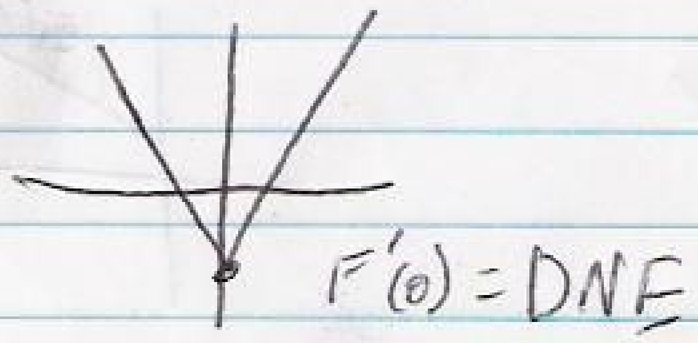
ex.3

$F(t) = |t| - 1$



ex. 4

$$\int_1^2 x^2\,dx = \frac{2^3}{3} - \frac{1^3}{3} = \frac{7}{3}$$

$$\left(\frac{x^3}{3}\right)' = x^2 \qquad (FTC_2)$$

ex. 5

r is rational constant & $r \neq -1$

$$\left(\frac{x^{r+1}}{r+1}\right)' = \left(\frac{1}{r+1} \cdot x^{r+1}\right) = \frac{1}{r+1}\left(x^{r+1}\right)$$

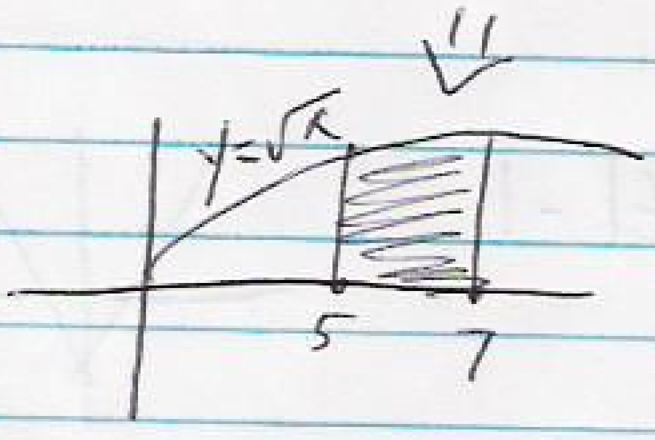
$$= \frac{1}{\cancel{(r+1)}}\cancel{(r+1)}\,x^{r+1-1} = x^r$$

$$\int_5^7 \sqrt{x}\,dx = \int_5^7 x^{1/2}\,dx = \frac{7^{1/2+1}}{1/2+1} - \frac{5^{1/2+1}}{1/2+1}$$

$$\frac{7^{3/2}}{3/2} - \frac{5^{3/2}}{3/2} = \frac{2}{3}\,7^{3/2} - \frac{2}{3}\cdot 5^{3/2}$$

$$\frac{2}{3}\left(7\sqrt{7} - 5\sqrt{5}\right)$$

$$\Downarrow$$



To Find $\int_a^b f(x)\,dx$, we usually need to solve $G'(x) = f(x)$ for $G(x)$

Then: $\int_a^b f(x)\,dx = G(b) - G(a) = \overbrace{G(x)\Big|_a^b}^{\text{abbreviation}}$

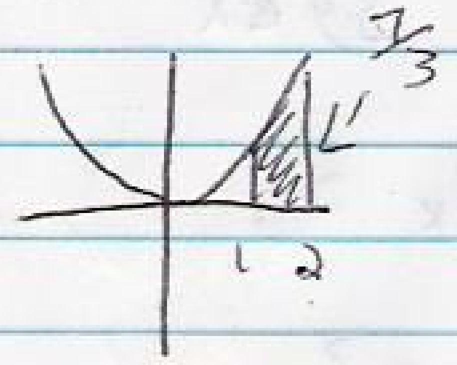
$\int_a^b f(x)\,dx$ is called a "definite integral" ← a number

no boundries → $\int f(x)\,dx$ is the general solution to $G'(x) = f(x)$ for $G(x)$

↑ Family of functions

an ~~definite integ~~ "indefinite integral" or "general atiderivative"

$$\int_1^2 x^2\,dx = \frac{7}{3}$$



$$\int x^2\,dx = \frac{x^3}{3} + C$$

$$\left(\frac{x^3}{3} + C\right)' = \frac{3x^2}{3} + 0 = x^2$$

refer to ex.5

$$(x^r)' = rx^{r-1}$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C$$

$r \neq -1$

r is constant

$\int x^r dx = \frac{x^{r+1}}{r+1} + C$	$(x^r)' = rx^{r-1}$	Power rule
$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$	$[f(x)+g(x)]' = f'(x) + g'(x)$	sum rule
$\int k f(x) dx = K \int f(x)$, k constant	$(K f(x))' = k \cdot f'(x)$	constant multiple rule

do not do

1. $\int f(x) g(x)\, dx \neq \int f(x) dx \int g(x) dx$, use power rule
2. $\int \frac{f}{g}\, dx \neq \frac{\int f\, dx}{\int g\, dx}$, use quotient rule
3. $\int f(g(x))\, dx = ?$, use chain rule

   $\int e^{(-x^2)}\, dx = ?$

ex. 6 $\int x^4 dx = \frac{x^5}{5} + C$

$\int x^2 dx \int x^2 dx = \left(\frac{x^3}{3} + C\right)\left(\frac{x^3}{3} + C_2\right) = \frac{x^6}{9} + (C_1 + C_2)\frac{x^3}{3} + C_1 C_2$

$\neq \int x^4 dx$

$\int x^2 dx \int x^2 dx \neq \int x^2 x^2 dx = \int x^4 dx$

ex. 7

$$\int_0^1 (5x^2 + 3x - 2)\,dx$$

$$= 5\int_0^1 x^2\,dx + 3\int_0^1 x\,dx - 2\int_0^1 dx$$

$$= 5\left(\frac{x^3}{3}\right)\Big|_0^1 + 3\left(\frac{x^2}{2}\right)\Big|_0^1 - 2\left(\frac{x^1}{1}\right)\Big|_0^1$$

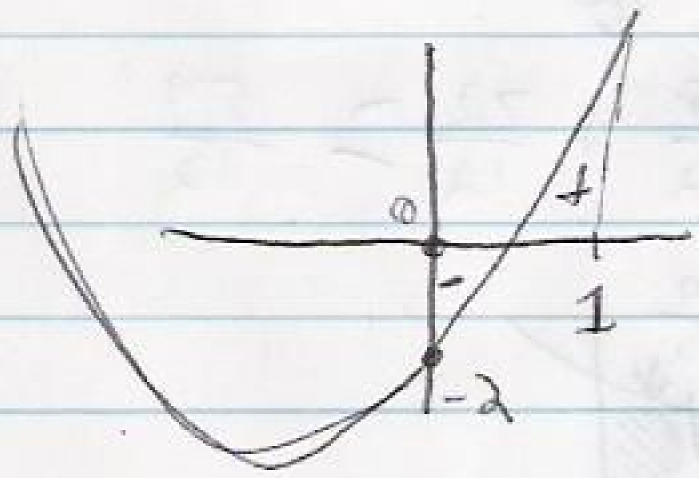
or

$$\left(\frac{5x^3}{3} + \frac{3x^2}{2} - 2x\right)\Big|_0^1$$

$$\left(\frac{5\cdot 1^3}{3} + \frac{3\cdot 1^2}{2} - 2\cdot 1\right) - \left(\frac{5\cdot 0^3}{3} + \frac{3\cdot 0^2}{2} - 2\cdot 0\right)$$

$$\frac{5}{3} + \frac{3}{2} - 2 - (0 + 0 - 0)$$

$$\frac{10}{6} + \frac{9}{6} - \frac{12}{6} = \frac{7}{6} = \text{area}$$



$y = 5x^2 + 3x - 2$

[redacted]

ex.8

$$\int_{-1}^{0} (x^2+3)(x+2)\,dx \neq \int (x^2+3x)\,dx \int (x+2)\,dx$$

$$= \int_{-1}^{0} (x^3+2x^2+3x+6)\,dx =$$

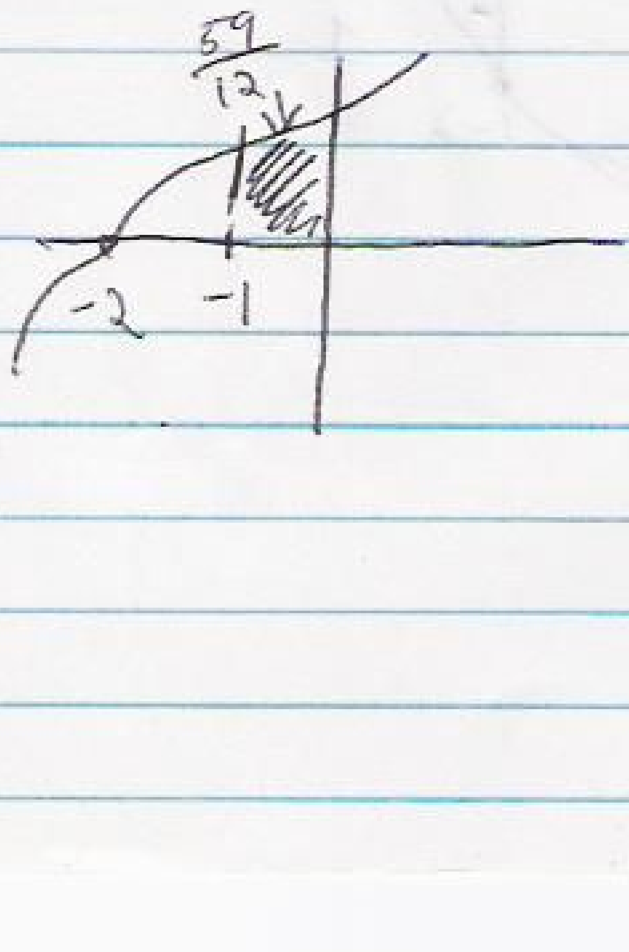
$$\left[\frac{x^4}{4} + 2\cdot\frac{x^3}{3} + 3\cdot\frac{x^2}{2} + \underbrace{6\cdot\frac{x^1}{1}}_{6x}\right]\Bigg|_{-1}^{0} =$$

$$\left(\frac{0^4}{4} + 2\cdot\frac{0^3}{3} + 3\cdot\frac{0^2}{2} + 6(0)\right) - \left(\frac{(-1)^4}{4} + 2\cdot\frac{(-1)^3}{3} + 3\frac{(-1)^2}{2} + 6\cdot(-1)\right)$$

$$= -\left(\frac{1}{4} - \frac{2}{3} + \frac{3}{2} - 6\right)$$

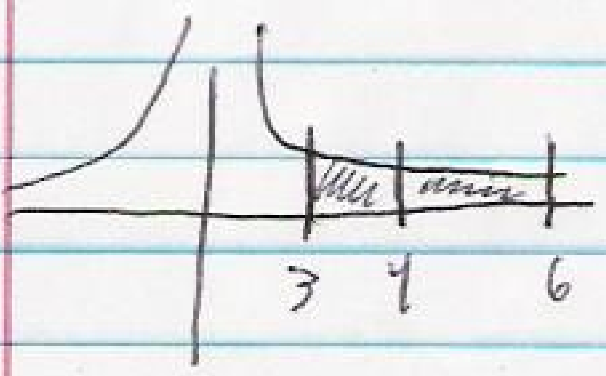
$$-\frac{1}{4} + \frac{2}{3} - \frac{3}{2} + 6$$

$$\frac{-3}{12} + \frac{8}{12} - \frac{18}{12} + \frac{72}{12} = \frac{59}{12}$$



ex. 9

$$\int_3^4 \frac{dx}{x^2} + \int_4^6 \frac{dx}{x^2} = \int_3^6 \frac{dx}{x^2} = \int_3^6 x^{-2}\,dx = \frac{x^{-2+1}}{-2+1}\Bigg|_3^6 \quad \text{O}$$



$$a<b<c \Rightarrow \int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\text{O} = \frac{x^{-1}}{-1}\Bigg|_3^6 = \left(-\frac{1}{x}\right)\Bigg|_3^6 = -\frac{1}{6} - \left(-\frac{1}{3}\right) = -\frac{1}{6} + \frac{1}{3} = \frac{1}{6}$$