Janette Chavez

Notes

$$r \neq -1 \Rightarrow \left(\frac{x^{r+1}}{r+1}\right)' = x^r \Rightarrow \int x^r dx = \frac{x^{r+1}}{r+1} + C \Rightarrow$$

$$\int_a^b x^r dx = \frac{x^{r+1}}{r+1}\Bigg|_a^b = \frac{b^{r+1}}{r+1} - \frac{a^{r+1}}{r+1}$$

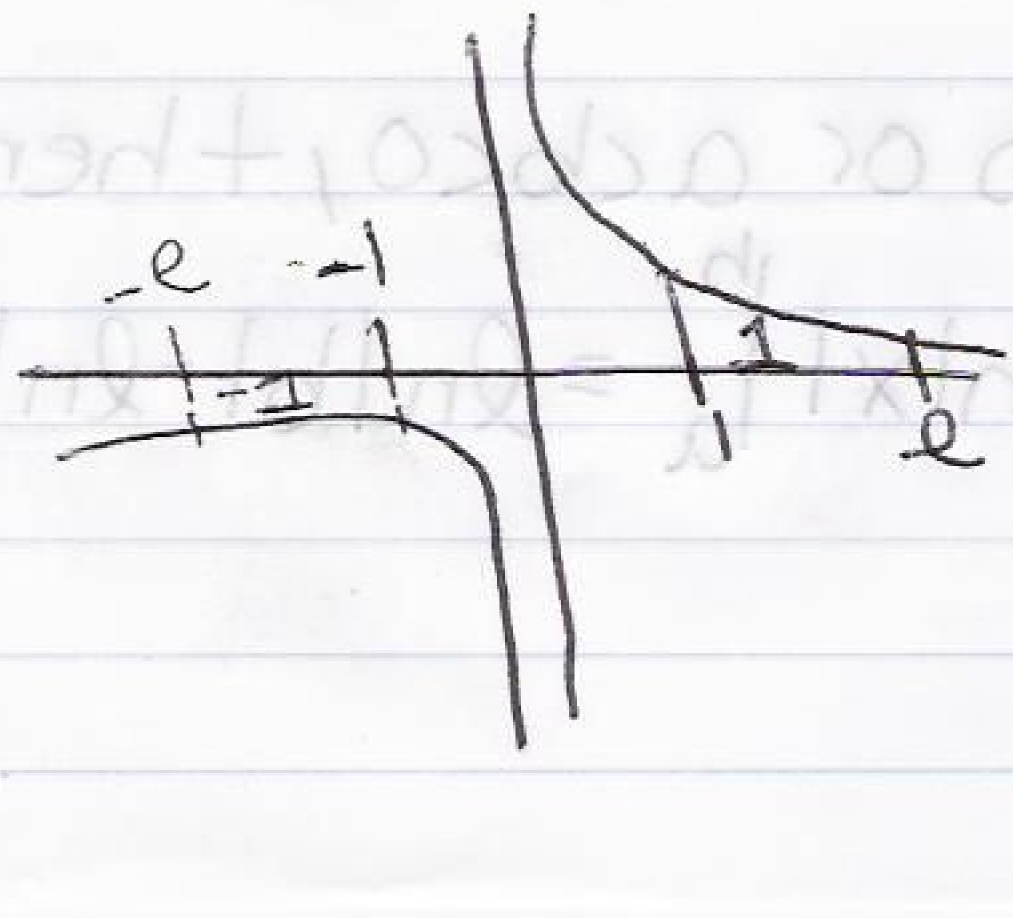
$$\int x^{-1} dx = ? \quad (\ln x)' = \frac{1}{x} \Rightarrow \int \frac{dx}{x} =$$

$\ln x$ only defined for $x > 0$.

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln x + C \Rightarrow$$

$$\int_1^e \frac{dx}{x} = \ln x \Big|_1^e = \ln e - \ln 1 = 1 - 0 = 1$$

works because
$0 < 1 < e$



If $x < 0$, then $-x > 0$, so $\ln(-x)$ is defined & $(\ln(-x))' \underset{\text{chain rule}}{=} \frac{1}{-x} \cdot (-x)' = \frac{1}{-x} \cdot -1 = \frac{1}{x}$

$x < 0 \Rightarrow (\ln(-x))' = \frac{1}{x} \Rightarrow \int \frac{dx}{x} = \ln(-x) + C \Rightarrow$

$$\int_{-e}^{-1} \frac{dx}{x} = \ln(-x)\Big|_{e}^{-1} = \ln(-(-1)) - \ln(-(-e))$$
$$= \ln 1 - \ln e$$
$$= 0 - 1 = -1$$

$$|x| = \begin{cases} x : x \geq 0 \\ -x : x < 0 \end{cases} \Rightarrow (\ln|x|)' = \begin{cases} (\ln x)' : x > 0 \\ \text{und} : x = 0 \\ (\ln(-x))' : x < 0 \end{cases}$$

$$= \begin{cases} 1/x : x > 0 \\ \text{DNE} : x = 0 \\ 1/x : x < 0 \end{cases} = \begin{cases} 1/x : x \neq 0 \\ \text{DNE} : x = 0 \end{cases}$$

$$(\ln|x|)' \underset{\text{when } x \neq 0}{=} \frac{1}{x} \Rightarrow \int \frac{dx}{x} = \ln|x| + C \Rightarrow$$

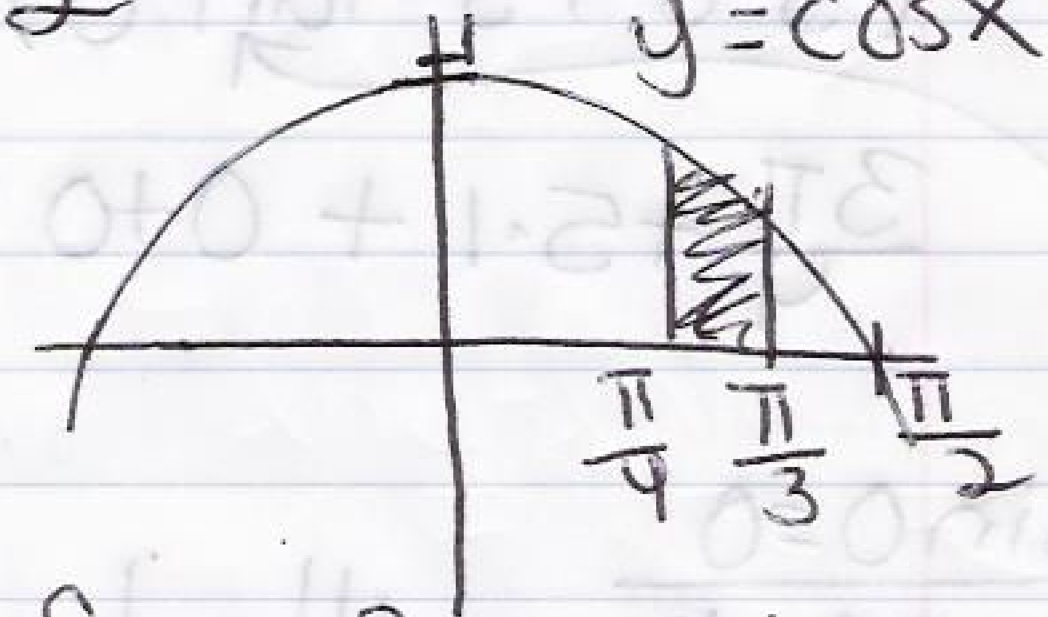
If $0 < a < b$ or $a < b < 0$, then

$$\int_a^b \frac{dx}{x} = \ln|x|\Big|_a^b = \ln|b| - \ln|a| = \ln\frac{|b|}{|a|}$$

$(-\cos x)' = -(-\sin x) = \sin x \Rightarrow$

$\int \sin x\,dx = -\cos x + C \Rightarrow \int_a^b \sin x\,dx = -\cos x \Big|_a^b =$
$(-\cos b) - (\cos a) = \cos a - \cos b$

$(\sin x)' = \cos x \Rightarrow \int \cos x\,dx = \sin x + C \Rightarrow$
$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos x\,dx = \sin x \Big|_{\pi/4}^{\frac{\pi}{3}} = \sin\frac{\pi}{3} - \sin\frac{\pi}{4} =$
$\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} = \frac{\sqrt{3}-\sqrt{2}}{2}$



$(\tan x)' = \sec^2 x \Rightarrow \int \sec^2 x\,dx = \tan x + C$
$(\sec x)' = \sec x \tan x \Rightarrow \int \sec x \tan x\,dx = \sec x + C$
$(\cot x)' = -\csc^2 x \Rightarrow \int -\csc^2 x\,dx = \cot x + C \Rightarrow$
$\int \csc^2 x\,dx = -\cot x + C$
$(\csc x)' = -\csc x \cot x \Rightarrow \int -\csc x \cot x\,dx =$
$\csc x + C \Rightarrow \int \csc x \cot x\,dx = -\csc x + C$

Avoid discontinuities:
$\sec x = \frac{1}{\cos x}$ + $\tan x = \frac{\sin x}{\cos x}$ are undefined

where $\cos x = 0$; at $\pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2}, \ldots$

$\csc x = \frac{1}{\sin x}$ + $\cot x = \frac{\cos x}{\sin x}$ are undefined where $\sin x = 0$: $0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \ldots$

$$\int_0^{\pi/4} \left(3 + \frac{5}{\cos^2 x}\right) dx = \int_0^{\pi/4} (3 + 5\sec^2 x)\, dx =$$

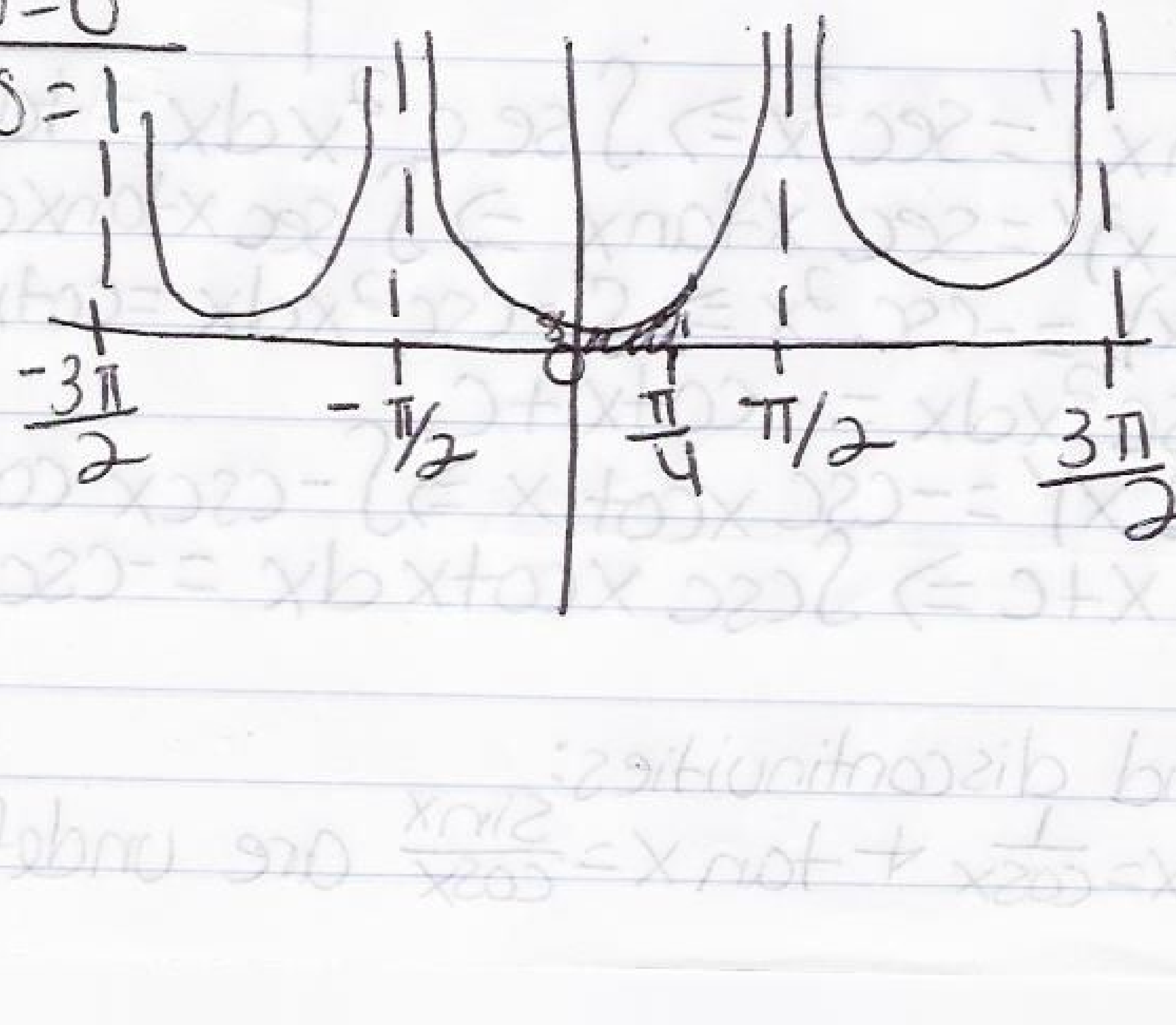
$$(3x + 5\tan x)\Big|_0^{\pi/4} = 3(\pi/4) + 5\underbrace{\tan\tfrac{\pi}{4}}_{\frac{\sin\frac{\pi}{4} = 1/\sqrt{2}}{\cos\frac{\pi}{4} = 1/\sqrt{2}}} -$$

$$(3 \cdot 0 + 5 \cdot \tan 0) =$$

$$\frac{3\pi}{4} + 5 \cdot 1 + 0 + 0 = \boxed{\frac{3\pi}{4} + 5}$$

$\frac{\sin 0 = 0}{\cos 0 = 1}$

$y = 3 + \frac{5}{\cos^2 x}$



$(e^x)' = e^x \Rightarrow \int e^x dx = e^x + C \Rightarrow$

$\int_{-1}^{1} 4e^x dx = 4e^x \Big|_{-1}^{1} = 4e^1 - 4e^{-1} = 4e - \frac{4}{e}$

$\int_{2}^{0} e^x dx = e^x \Big|_{2}^{0} = e^0 - e^2 = 1 - e^2$

In general, $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
when $a<b$ (and when $a \geq b$ too)

Recall that $\int_a^a f(x)\,dx = 0$.

$\int \frac{x^7 + \sqrt{x} - 3x + 2}{x} dx = \int (x^6 + \underbrace{x^{-\frac{1}{2}}}_{\frac{\sqrt{x}}{x} = \frac{x^{1/2}}{x^1}} - 3 + \underbrace{2x^{-1}}_{\frac{2}{x}})dx =$

$= \frac{x^{6+1}}{6+1} + \frac{x^{-1/2+1}}{-\frac{1}{2}+1} - 3x + 2\ln|x| + c$

$= \frac{x^7}{7} + 2\sqrt{x} - 3x + 2\ln|x| + C$

Over 60 minutes - from $t=0$ to $t=60$ - your velocity at time t is $15 + t/3 - t^2/1000$ mph, directed west. How far do you travel in those 60 minutes?

$$\int_0^{60\,\text{min}} \underbrace{\left(15+\frac{t}{3}-\frac{t^2}{100}\right)}_{\text{miles/hour}} dt = \underbrace{15t+\frac{1}{3}\left(\frac{t^2}{2}\right)-\frac{1}{1000}\left(\frac{t^3}{3}\right)}_{\text{miles}}$$

$$= \left(15t+\frac{1}{3}\left(\frac{t^2}{2}\right)-\frac{1}{1{,}000}\left(\frac{t^3}{3}\right)\right)^{60\,\text{min}}_{0} =$$

$$\left(15\cdot 60+\frac{1}{3}\cdot\frac{60^2}{2}-\frac{1}{1000}\cdot\frac{60^3}{3}\right)\cdot\frac{1}{60}\text{ miles} = (15+10-1.2)\text{ miles} =$$

$$\boxed{23.8 \text{ miles}}$$