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Notes Calculus 11/17/2010

x = displacement = position relative to some fixed point
$v = dx/dt$ = velocity
$a = dv/dt = d^2x/dt$ = acceleration

Suppose you know the acceleration $a(t)$ at all times
velocity $v(t_1)$ at time t_1
and position $x(t_2)$ at time t_2

We can compute $v(t)$ & $x(t)$ for all t:

$$\int_{t_1}^{t} a(T)\,dT = v(t) - v(t_1) \Rightarrow \boxed{v(t_1) + \int_{t_1}^{t} a(T)\,dT = v(t)}$$

because $v'(t) = a(t)$

$$\Downarrow$$

$$\int a(T)\,dT = v(T) + c$$

repeat Pattern

$$x(t_2) + \int_{t_2}^{t} v(T)\,dT = x(t)$$

$x(t)$ = height above ground at time t

Vert. accel. $\hookrightarrow a(t) = -g = -9.8\,m/s^2 = -32\,ft/s^2$

$v(t)$ = vertical velocity

$v(0) = 10\,ft$

$x(0) = 6\,ft$

valid when in midair for all t in $(0, t_{impact})$

$$v(0) + \int_0^t (-32\,ft/s^2)\,dT = v(t)$$

$$10\,ft/s \; -32(ft/s^2)T \Big|_0^t = 10\,ft/s^2 - (32t - 32\cdot 0)\,ft/s^2$$

$$10\,ft/s - (32\,ft/s^2)t = v(0) - gt = v(t)$$

$$x(0) + \int_0^t v(T)\,dT = x(t)$$

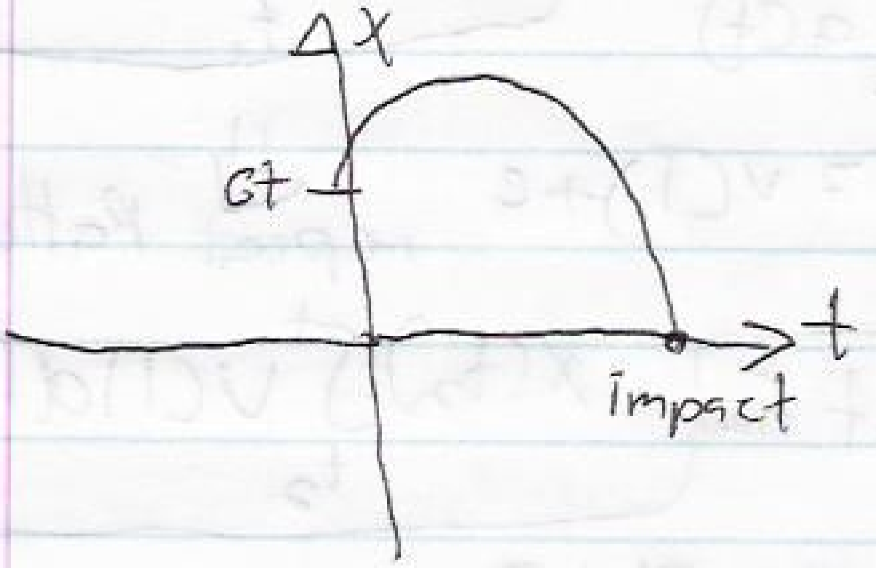
$$x(0) + \int_0^t (v(0) - gT)\,dT$$

$$x(0) + \left(v(0)T - \tfrac{1}{2}gT^2\right)\Big|_0^t$$

$$x(0) + \left(v(0)t - \tfrac{1}{2}gt^2\right) - \left(v(0)\cdot 0 - \tfrac{1}{2}g\cdot(0)^2\right)$$

$$0 - 0$$

$$x(t) = x(0) + v(0)t - \tfrac{1}{2}gt^2 = 6ft + (10ft/s)t - \tfrac{1}{2}(32\,ft/s^2)t^2$$



Suppose $a(t) = k\cos(t)$

$v(0) = 2m/s$ $\quad k = 3m/s^2$ $\quad$ t measure in secs

$x(0) = 0m$

$$V(t) = v(0) + \int_0^t a(T)\,dT$$

$$v(0) + \int_0^t k\cos T\,dT$$

$$v(0) + (k\sin T)\Big|_0^t = v(0) + k\sin t - k\sin 0$$

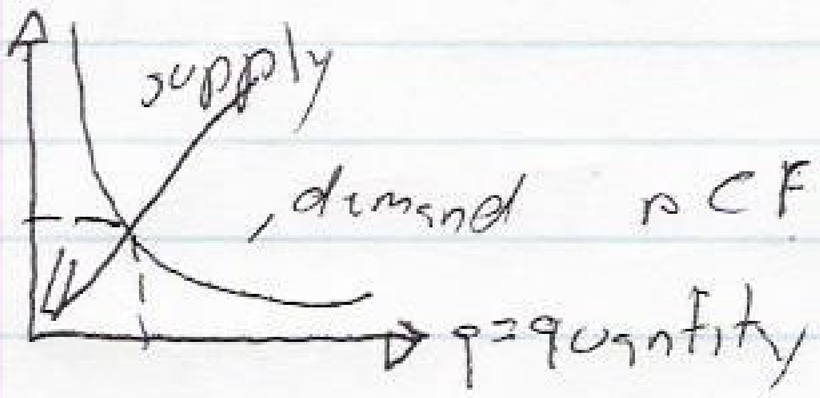
$$v(0) + k\sin t$$

$x(t) = x(0) + \int_0^t v(\tau)\,d\tau = x(t) = 0 + \int_0^t (v(0) + k\sin k\tau)\,d\tau$

$= v(0)\tau - k\cos\tau)\,d\tau \Big|_0^t = (v(0)t - k\cos t) - (v(0)\cdot 0 - \underbrace{k\cos 0}_{-k})$

$v(0)t - k\cos t + k$

p=price



$p = f(q) = \frac{15,000}{q}$

$p = g(q) = q/10$ (same as $q = 10p$

$\int_0^{q_{eq}} p_{eq} - g(q))$ = $f(q_{eq}) = p_{eq} = g(q_{eq})$

higher lower

$\frac{15,000}{q_{eq}} - \frac{q_{eq}}{10} \Rightarrow 150,000 = q_{eq}^2$

$P_{eq} = g(q_{eq}) = q_{eq}/10 = 100\sqrt{15}/10 \leftarrow \sqrt{150,000} = 100\sqrt{15} \sim 387$

$10\sqrt{15} \sim 387$

$\int_0^{100\sqrt{15}} (10\sqrt{15} - \frac{1}{10}q)\,dq = (10\sqrt{15}\,q - \frac{1}{10}(\frac{1}{2}q^2))\Big|_0^{100\sqrt{15}}$

$(10\sqrt{15}\cdot 100\sqrt{15} - \frac{1}{10}(\frac{1}{2}(100\sqrt{15})^2) - (10\sqrt{15}\cdot 0 - \frac{1}{10}(\frac{1}{2}\cdot 0^2)) = 7,500$

$15,000 - 7,500$ $\quad - \quad 0$