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11-18-10

Substitution (in Integrals)

$\int \sin \underbrace{u}_{x^2} \, dx = ?$

- $\int \sin(x^2)\,dx = ?$ No formula.

  $(\sin(x^2))' \overset{\text{chain rule}}{=} \cos(x^2)\cdot(2x)$

  $\sin(x^2)+C = \int \cos(x^2)\cdot\underbrace{(2x)\,dx}_{du} = \int \cos u \, du = \sin u + C = \boxed{\sin(x^2)+C}$

  Generalize:

  $d(\sin(x^2)) = (\sin(x^2))'\,dx = \cos(x^2)(2x)\,dx$

  $u = x^2 \Rightarrow d(\sin(x^2)) = d(\sin u) = (\sin u)'\,du = \underbrace{\cos u}_{\cos(x^2)}\,\overbrace{du}^{2x\,dx}$

- $\int \cos(3x)\,dx = \frac{1}{3}\cdot 3\cdot\int \cos(3x)\,dx = \frac{1}{3}\int \cos\underbrace{(3x)}_{u}\underbrace{(3\,dx)}_{du} = \frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u + C = \boxed{\frac{1}{3}\sin(3x)+C}$

  constant multiple

  $u = 3x$

  $du = (3x)'\,dx = 3\,dx$

- $\int e^{7t}\,dt = \int e^u \cdot \frac{du}{7} = \frac{1}{7}e^u + C = \boxed{\frac{1}{7}e^{7t}+C}$

  $u = 7t$

  $du = 7\,dt \Rightarrow \frac{du}{7} = dt$

  Check: $\left(\frac{1}{7}e^{7t}+C\right)' = \frac{1}{7}e^{7t}\overbrace{(7t)'}^{7} + 0 = e^{7t}$ ✓

- $\int 3^x\,dx = \int e^{(\ln 3)\cdot x} = \int e^u \frac{du}{\ln 3} = \frac{e^u}{\ln 3} + C = \frac{e^{(\ln 3)\cdot x}}{\ln 3} + C = \boxed{\frac{3^x}{\ln 3} + C}$

  $a^b = \underbrace{(e^{(\ln a)})}_{a}{}^b = e^{((\ln a)\cdot b)}$

  $u = (\ln 3)\cdot x$

  $du = (\ln 3)\,dx \Rightarrow \frac{du}{\ln 3} = dx$

  Check: $\left(\frac{3^x}{\ln 3} + C\right)' = \frac{3^x \ln 3}{\ln 3} + 0 = 3^x$ ✓

- $\int \tan x\,dx = \int \frac{\sec x \tan x\,dx}{\sec x} = \int \frac{du}{u} = \int u^{-1}\,du = \ln|u| + C = \boxed{\ln|\sec x| + C}$

  $u = \sec x$

  $du = (\sec x)'\,dx$

  $du = \sec x \tan x\,dx$

$\ln|\sec x| = \ln\left|\frac{1}{\cos x}\right|$

They're the same; $-\ln|\cos x| = (-1)\ln|\cos x| = \ln\left|(\cos x)^{-1}\right| = \ln\left|(\cos x)^{-1}\right|$

- Alternate solution method;

$$\int \tan x\,dx = \int \frac{\sin x\,dx}{\cos x} = \int \frac{-du}{u} = -\int \frac{du}{u} = -\ln|u| + c = \boxed{-\ln|\cos x| + C}$$

$u = \cos x$

$du = (\cos x)'\,dx$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

- $$\int \cot x\,dx = \int \frac{\cos x\,dx}{\sin x} = \int \frac{du}{u} = \ln|u| + c = \boxed{\ln|\sin x| + c}$$

$u = \sin x$

$du = (\sin x)'\,dx$

$du = \cos x\,dx$

$$\int \frac{f'(x)}{f(x)}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln|f(x)| + C$$

$u = f(x)$

$du = f'(x)\,dx$

- $$\int \underbrace{(2x+3)^{25}}\,dx = \int u^{25}\,\frac{du}{2} = \frac{1}{2}\int u^{25}\,du = \frac{1}{2}\cdot\frac{u^{26}}{26} + c = \frac{1}{2}\cdot\frac{(2x+3)^{26}}{26} + c = \boxed{\frac{(2x+3)^{26}}{52} + c}$$

hard way — $(2^{25}x^{25} + \underbrace{\ldots}_{\text{24 terms}} + 3^{25})$

easy way :) — $u = 2x+3$

$du = 2dx$

$\frac{du}{2} = dx$

- $\int x^2\sqrt{x^3+5}\,dx = \int \sqrt{u}\cdot\frac{du}{3} = \frac{1}{3}\int\sqrt{u}\,du = \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\,\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{1}{3}\cdot\frac{u^{3/2}}{3/2} + C$

$u = x^3+5$

$du = 3x^2\,dx$ $\qquad = \frac{1}{3}\cdot\frac{2}{3}\cdot u^{3/2} + C = \frac{2}{9}u^{3/2} + C = \boxed{\frac{2}{9}(x^3+5)^{3/2} + C}$

$\frac{du}{3} = x^2$

- $\int \frac{x^3\,dx}{x^4 \ln x} = \int \frac{du/4}{u\ln(u^{1/4})} = \int \frac{du/4}{u(\ln u)/4} = \int \frac{du}{u\ln u}$

1. $u = x^4 \Rightarrow x = \sqrt[4]{u} = u^{1/4}$

$du = 4x^3 dx$ $\qquad = \int \frac{dx}{x\ln x} = \int \frac{1}{\ln x}\cdot\frac{dx}{x} = \int\frac{1}{w}\,dw = \ln|w| + C = \boxed{\ln|\ln x| + C}$

$\frac{du}{4} = x^3\,dx$

2. $w = \ln x$

$dw = (\ln x)'$

$dx = \frac{1}{x}\,dx = \frac{dx}{x}$