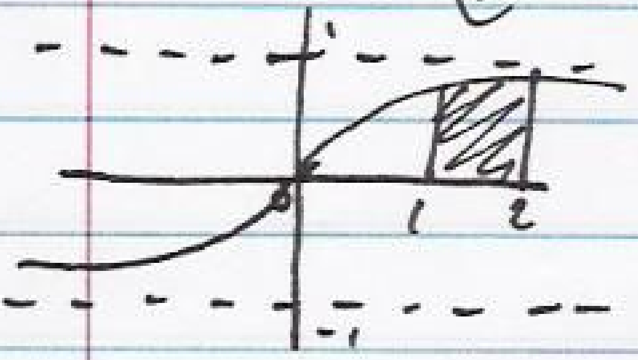
Nov 22, 2010 Diego

"u" substitution with definate integrals

$\int_1^2 \frac{x\,dx}{\sqrt{x^2+1}}$

$y = \frac{x}{\sqrt{x^2+1}}$



First, try to find $\int \frac{x\,dx}{\sqrt{x^2+1}}$, then plug in 2 & 1, and take the diff.

$$\int \frac{x\,dx}{\sqrt{x^2+1}} = \int \frac{du/2}{\sqrt{u}} = \int \boxed{\frac{1}{2}\frac{1}{\sqrt{u}}}\,du = \int \frac{1}{2}u^{-1/2}\,du = \frac{1}{2}\int u^{-1/2}\,du =$$

$u = x^2+1$

$du = (x^2+1)'\,dx$

$du = 2x\,dx$

$du/2 = x\,dx$

$$\frac{1}{2}\cdot\frac{u^{-1/2+1}}{-1/2+1} + C = \frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + C = u^{1/2} + C = \sqrt{u} + C = \sqrt{x^2+1} + C$$

$$\int_1^2 \frac{x\,dx}{\sqrt{x^2+1}} = \sqrt{x^2+1}\,\Big|_1^2 = \sqrt{2^2+1} - \sqrt{1^2+1} = \boxed{\sqrt{5}-\sqrt{2}}$$

· Alternative solution:

$$\int_1^2 \frac{x\,dx}{\sqrt{x^2+1}} = \int_2^5 \frac{du/2}{\sqrt{u}} = \frac{1}{2}\int_2^5 u^{-1/2}\,du$$

$$= \frac{1}{2}\cdot\frac{u^{-1/2+1}}{-1/2+1}\Big|_2^5 = \frac{1}{2}\cdot\frac{u^{1/2}}{1/2}\Big|_2^5 = \sqrt{u}\,\Big|_2^5 = \boxed{\sqrt{5}-\sqrt{2}}$$

$u = x^2+1$

$du = (x^2+1)'\,dx = 2x\,dx$

$du/2 = x\,dx$

$x = 1 \Rightarrow u = 1^2+1 = 2$

$x = 2 \Rightarrow u = 2^2+1 = 5$

$$\int_0^1 x^3\sqrt[4]{5x^2+2}\,dx = \int_0^1 \underbrace{x^2}_{?}\,\underbrace{\sqrt[4]{5x^2+2}}_{\sqrt[4]{u}}\,\underbrace{x\,dx}_{du/10} = \int_2^7 \left(\frac{u-2}{5}\right)u^{1/4}\cdot\frac{du}{10} = \text{[illegible]}$$

$$= \frac{1}{50}\int_5^7 (u-2)u^{1/4}\,du = \frac{1}{50}\int_5^7 (u^{5/4} - 2u^{1/4})\,du = \frac{1}{50}\left(\frac{u^{5/4+1}}{5/4+1} - 2\frac{u^{1/4-1}}{1/4-1}\right)\Big|_2^7$$

$$= \frac{1}{50}\left(\frac{u^{9/4}}{9/4} - 2\frac{u^{5/4}}{5/4}\right)\Big|_2^7 = \frac{1}{50}\left(\frac{4}{9}u^{9/4} - \frac{8}{5}u^{5/4}\right)\Big|_2^7 = \frac{1}{50}\left[\left(\frac{4}{9}\cdot 7^{9/4} - \frac{8}{5}\cdot 7^{5/4}\right) - \left(\frac{4}{9}\cdot 2^{9/4} - \frac{8}{5}\cdot 2^{5/4}\right)\right]$$

$u = 5x^2+2$

$du = (5x^2+2)'\,dx$

$= 10x\,dx$

$du/10 = x\,dx$

$u - 2 = 5x^2$

$\frac{u-2}{5} = x^2$

$x = 0 \Rightarrow u = 5\cdot 0^2 + 2 = 2$

$x = 1 \Rightarrow u = 5\cdot 1^2 + 2 = 7$

$\int_{-1}^{0} \frac{e^{-3x}\,dx}{(e^{-3x}-5)^2}$ is not defined because $-1 < -\frac{\ln 5}{3} < 0$ &

$x = -\frac{\ln 5}{3} \Rightarrow e^{-3x} - 5 = 0$, so you have division by 0 and a discontinuity.

$u = e^{-3x} - 5$

$du = (e^{-3x}-5)'\,dx$

$du = (e^{-3x}(-3x)' - 0)\,dx$

$du = -3e^{-3x}\,dx$

$-\frac{du}{3} = e^{-3x}\,dx$

$x=0 \Rightarrow u = e^{-3\cdot 0} - 5 = e^0 - 5 = 1-5 = -4$

$x=+1 \Rightarrow u = e^{-3\cdot 1} - 5 = e^{-3} - 5$

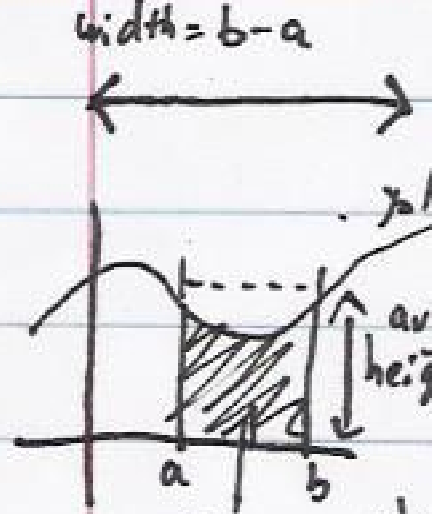
$$\int_{+1}^{0} \frac{e^{-3x}\,dx}{(e^{-3x}-5)^2} = \int_{e^{-3}-5}^{-4} \frac{-du/3}{u^2} = \int_{-4}^{e^{-3}-5} \frac{du/3}{u^2} = \frac{1}{3}\int_{-4}^{e^{-3}-5} u^{-2}\,du = \frac{1}{3}\cdot\frac{u^{-2+1}}{-2+1}\Bigg|_{-4}^{e^{-3}-5}$$

$$= \frac{1}{3}\cdot\frac{u^{-1}}{-1}\Bigg|_{-4}^{e^{-3}-5} = \frac{-1}{3u}\Bigg|_{-4}^{e^{-3}-5} = \frac{1}{3u}\Bigg|_{e^{-3}-5}^{-4} = \boxed{\frac{1}{3(-4)} - \frac{1}{3(e^{-3}-5)}}$$

$$\int_a^b -f(u)\,du = \int_b^a f(u)\,du \qquad (-F(d)) - (-F(c)) = -F(u)\Big|_c^d$$

$$= F(c) - F(d) = F(u)\Big|_d^c$$

★ Continuous Averages: The avg. of $f(x)$ over $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$ is $\frac{1}{\text{width}}\cdot\text{area}$ is avg. height



avg. height $= \frac{1}{b-a}\int_a^b f(x)\,dx$

area $= \int_a^b f(x)\,dx$

· An obj. is dropped from 4 ft. high. What is its avg. speed during its fall? Assume acceleration = 32 ft/s². $a = \frac{dv}{dt} = 32\ \text{ft/s}^2 = g$

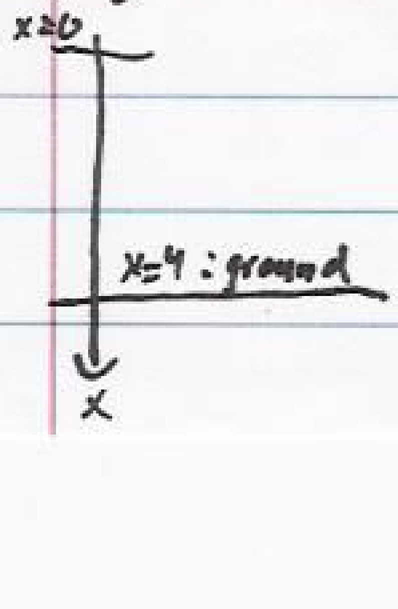
$v(t) = v(0) + \int_0^t g\,dT$

$v(t) = 0 + gT\Big|_0^t = gt$

$x(t) = x(0) + \int_0^t v(T)\,dT$

$x(t) = 0 + \int_0^t gT^2\Big|_0^t = \frac{1}{2}gt^2$

$\sqrt{8/g} = t$ when hits ground



$$\text{Avg. speed} = \frac{1}{\sqrt{8/g}-0}\int_0^{\sqrt{8/g}} v(T)\,dT = \frac{1}{\sqrt{8/g}}\cdot\frac{1}{2}gT^2\Bigg|_0^{\sqrt{8/g}}$$

$$= \frac{1}{\sqrt{8/g}}\cdot\frac{1}{2}g\left(\frac{\sqrt{8}}{\sqrt{g}}\right)^2 = \frac{1}{\sqrt{8/g}}\cdot\frac{1}{2}g\cdot\frac{8}{g} = \frac{4}{\sqrt{8/g}} = \frac{\text{distance}}{\text{time}}$$