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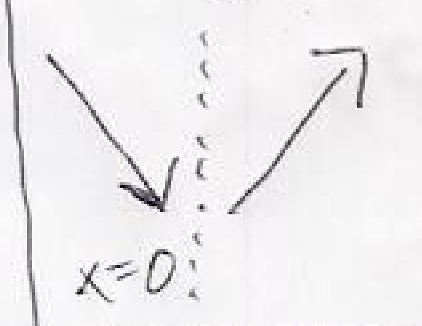
Crude graph



$x>0 \Rightarrow x^3>0 \Rightarrow 2/x^3>0$
$x<0 \Rightarrow x^3<0 \Rightarrow 2/x^3<0$
f is $\nearrow$ on $(0,\infty)$
f is $\searrow$ on $(-\infty,0)$

$y=f(x)=e^{-1/x^2}=e^{-x^{-2}}$

Find maximal intervals where f is $\nearrow$, $\searrow$, CU, CD
Find horizontal & vertical asymptotes.

f	f'	f''
$\nearrow$	$+$	
$\searrow$	$-$	
CU	$\nearrow$	$+$
CD	$\searrow$	$-$

$u=-1/x^2=-x^{-2}$

$f'=\frac{df}{dx}=\frac{df}{du}\frac{du}{dx}=\boxed{e^{u}\cdot 2/x^3=\underbrace{e^{(-1/x^2)}}_{e^{\text{anything}}>0}\cdot(2/x^3)=f'(x)}$

$f=e^{-1/x^2}=e^{u}$

$\frac{df}{du}=(e^u)'=e^u \quad \frac{du}{dx}=-(-2)x^{-2-1}=2x^{-3}=2/x^3$

Cannot extend $(0,\infty)$ to $[0,\infty)$ & still have $f\nearrow$ on $[0,\infty)$ because $f(0)=e^{-1/0^2}$ is undefined. Likewise, we can't extend $(-\infty,0)$ to $(-\infty,0]$.

Recall: $f\nearrow$ on I means for all $x_1<x_2$ in I, $f(x_1)<f(x_2)$

$f\nearrow$ on $(0,\infty)$ | $f'=(e^{(-1/x^2)})(2/x^3)$
$f\searrow$ on $(-\infty,0)$

$f'=2x^{-3}e^{-x^{-2}}$

$f''=(2x^{-3}e^{-x^{-2}})'$

$=\underset{\substack{\uparrow\\ \text{Product Rule}}}{(2x^{-3})'}e^{-x^{-2}}+2x^{-3}\underbrace{(e^{-x^{-2}})'}_{f'}=2(-3)x^{-3-1}e^{-x^{-2}}+2x^{-3}(2x^{-3}e^{-x^{-2}})$

$=-6x^{-4}e^{-x^{-2}}+4x^{-6}e^{-x^{-2}}$

$=\underbrace{(-6x^{-4}+4x^{-6})}_{f''}\underbrace{e^{-x^{-2}}}_{\text{always positive}}$

$\longrightarrow$

$-6x^{-4}+4x^{-6} = \frac{-6}{x^4}+\frac{4}{x^6} = \frac{-6x^2}{x^4 \cdot x^2}+\frac{4}{x^6} = \frac{-6x^2+4}{x^6}$ ← make sign table...

$\frac{-6x^2+4}{x^6}$ is undefined at $x^6=\emptyset$, at $\boxed{x=\emptyset}$

$\frac{-6x^2+4}{x^6}=\emptyset \Rightarrow -6x^2+4=\emptyset \cdot x^6=\emptyset$

$-6x^2+4=\emptyset \Rightarrow 4=6x^2 \Rightarrow \frac{2}{3}=x^2 \Rightarrow \boxed{x=\pm\sqrt{2/3}}$

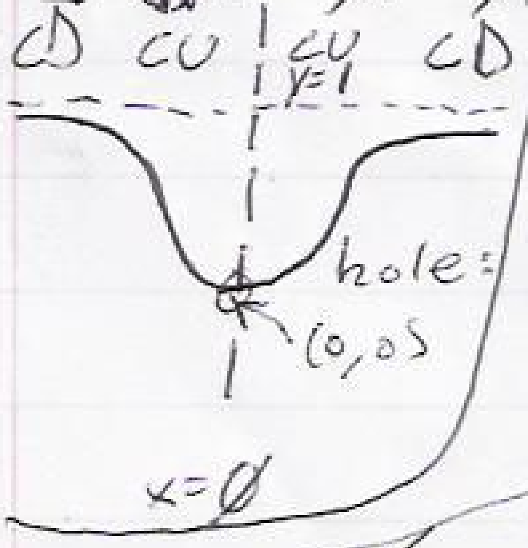
	test		test		test		test
x	-1	$-\sqrt{2/3}$	$-\frac{1}{2}$	$\emptyset$	$\frac{1}{2}=\sqrt{\frac{1}{4}}$	$\sqrt{2/3}$	1
$\frac{-6x^2+4}{x^6}$	(-2) - - - -	$\emptyset$ +++	(60) ++	ND +++	(60) ++	$\emptyset$ - - -	(-2) - - -
$f''=\frac{-6x^2+4}{x^6}(\exp(-x^{-2}))$	- - - - - - -	0 + + +	+ +	ND	+ + + + +	0 - - -	- - - - - -
f	CD	(I.P.)	CU	ND	CU	(I.P.)	CD

Max Int.

$\underbrace{(-\infty,-\sqrt{2/3}]}_{f \text{ is CD}}$ $\underbrace{[-\sqrt{2/3},0)}_{f \text{ is CU}}$ $\underbrace{(0,\sqrt{2/3}]}_{f \text{ is CU}}$ $\underbrace{[\sqrt{2/3},\infty)}_{f \text{ is CD}}$

(vertical asymptote = Infinite Limit)

Less Crude "graph"



H.A.s: $\lim_{x\to\infty} e^{-1/x^2} = 1$ → Let H be pos. inf. [redacted] sorry :)

$\lim_{x\to-\infty} e^{-1/x^2} = 1$ → plug in $-H$

$e^{\overbrace{-1/H^2}^{-\text{small}\approx\emptyset}} \approx e^{\emptyset}=1$

$\boxed{y=1 \text{ is } \boxed{\text{only}} \text{ HA}}$

V.A.s: Can happen only at discontinuities, if at all

check $\lim_{x\to\emptyset^-} e^{-1/x^2}$ $\lim_{x\to\emptyset^+} e^{-1/x^2}$ ← both $=\emptyset$

Let ε be positive infinitesimal.

$e^{(-1/(-\varepsilon)^2)} \approx \emptyset$

$-1/(-\varepsilon)^2 = -1/\varepsilon^2 = -\text{med}/(+\text{small}) \approx -\text{big}$

Similarly, $e^{(-1/(\varepsilon^2))} \approx \emptyset$

$e^{-\text{big}} = +\text{small} \approx \emptyset$

$\boxed{\text{NO V.A.}}$ because $\boxed{\text{limits}}$ are not $\pm\infty$.