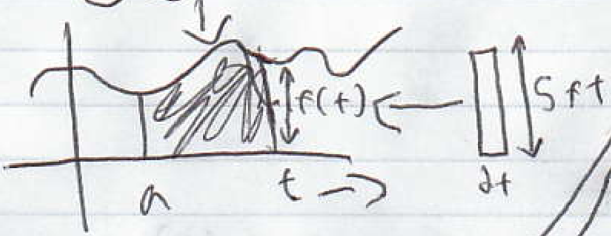
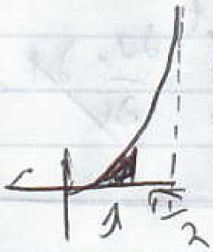
$$FTC_1 \quad \left(\int_a^t f(x)\,dx\right)' = f(t) \iff d\left(\int_a^t f(x)\,dx\right) = f(t)\,dt$$



$$\left(\int_0^{\sin(t)} \sqrt{\tan x}\,dx\right)' = ?$$

use chain rule



$$y = \int_0^{\sin(t)} \sqrt{\tan x}\,dx$$

$$u = \sin(t)$$

$$y = \int_0^u \sqrt{\tan x}\,dx$$

$$\boxed{\frac{dy}{dt} = \frac{dy}{du}\cdot\frac{du}{dt}}$$

just different letters

chain rule

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

$$\frac{dy}{du} = \frac{d\left(\int_0^u \sqrt{\tan x}\,dx\right)}{du} = \frac{\sqrt{\tan u}\,du}{du}$$

$$\frac{du}{dt} = (\sin(t))' = \cos(t)$$

$$\sqrt{\tan(\sin(t))}\ \cos t$$

$$\|$$

$$\sqrt{\tan u}\ \cos t$$

$$f(t) = a_x = \frac{dv_x}{dt}$$

$$? = v_x = \frac{dx}{dt}$$

$$? = x$$

$$p = v_x(0)$$

$$q = x(0)$$

$$\text{FTC}_1\quad d\left(\int_a^t f(x)\,dx\right) = f(t)\,dt$$

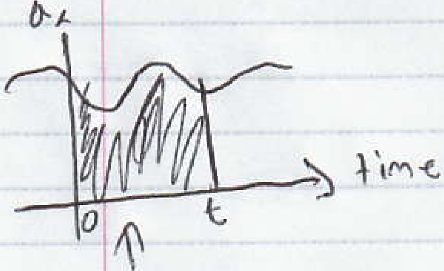
$$\text{FTC}_2\quad \int_\alpha^\beta g'(t)\,dt = g(\beta) - g(\alpha) \qquad \int_\alpha^\beta dg = g(\beta) - g(\alpha)$$

$$f(t) = a_x = \frac{dv_x}{dt}$$

$$\int_0^t f(t)\,dt = \int_0^t \frac{dv_x}{\cancel{dt}}\,\cancel{dt} = \int_0^t dv_x = v_x(t) - v_x(0) = v_x(t) - p$$

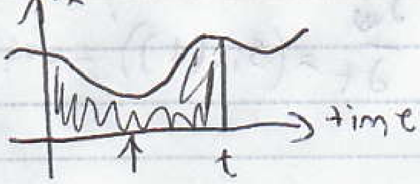
$$\boxed{v_x(t) = p + \int_0^t f(t)\,dt} \longrightarrow \int_0^t v_x\,dt = \int_0^t \frac{dx}{\cancel{dt}}\,\cancel{dt} = \int_0^t dx = x(t) - x(0)$$

$$x(t) - q \implies \boxed{x(t) = q + \int_0^t v_x\,dt}$$



area $= v_x(t) - v_x(0)$

$v_x(0) +$ area $= v_x(t)$



area $= x(t) - x(0)$

$x(0) +$ area $= x(t)$

If the population of Utopia is growing at a rate of 500 people/year (right now) and the ratio of Utopia's population to Rohan's population is decreasing at a rate of 0.007 per year, and Rohan's population is currently 90,000, and Utopia's population is currently 130,000, then what is the rate of change of Rohan's population?

Right now:

$U = 130000$

$R = 90000$

$\frac{dU}{dt} = 500$

$-7 \cdot 10^{-3} = \frac{d}{dt}\left(\frac{U}{R}\right) = -0.007 = \frac{U'R - UR'}{R^2} = \frac{\frac{dU}{dt} R - U\frac{dR}{dt}}{R^2}$

$\frac{dR}{dt} = ?$

$5 \cdot 10^2 = 500$

$130000 = 13 \cdot 10^4$

90000

$9 \cdot 10^4$

$\frac{dU}{dt} R = 45 \cdot 10^6 \qquad R^2 = 81 \cdot 10^8$

$\frac{45 \cdot 10^6 - 13 \cdot 10^4 \frac{dR}{dt}}{81 \cdot 10^8} = -7 \cdot 10^{-3}$

$45 \cdot 10^6 - 13 \cdot 10^4 \frac{dR}{dt} = -567 \cdot 10^5$

$$-13\cdot10^{4}\frac{dR}{dt} = -1017\cdot10^{5}$$

$$\frac{dR}{dt} = 782.3$$

782.3 people/year

$$\frac{d}{dx}\left(e^{\sqrt{x^4\sin(x^3)+1}}\sin\left(\frac{1}{\ln x}\right)\right)$$