The bet:
between 49% & 51%
of 1000 coin flips
will be heads.

From statistics, we know that for N coin flips, where N is large,

$$\left[\begin{array}{l}\text{Probability of} \\ a \leq \#\text{heads} - \#\text{tails} \leq b\end{array}\right]$$

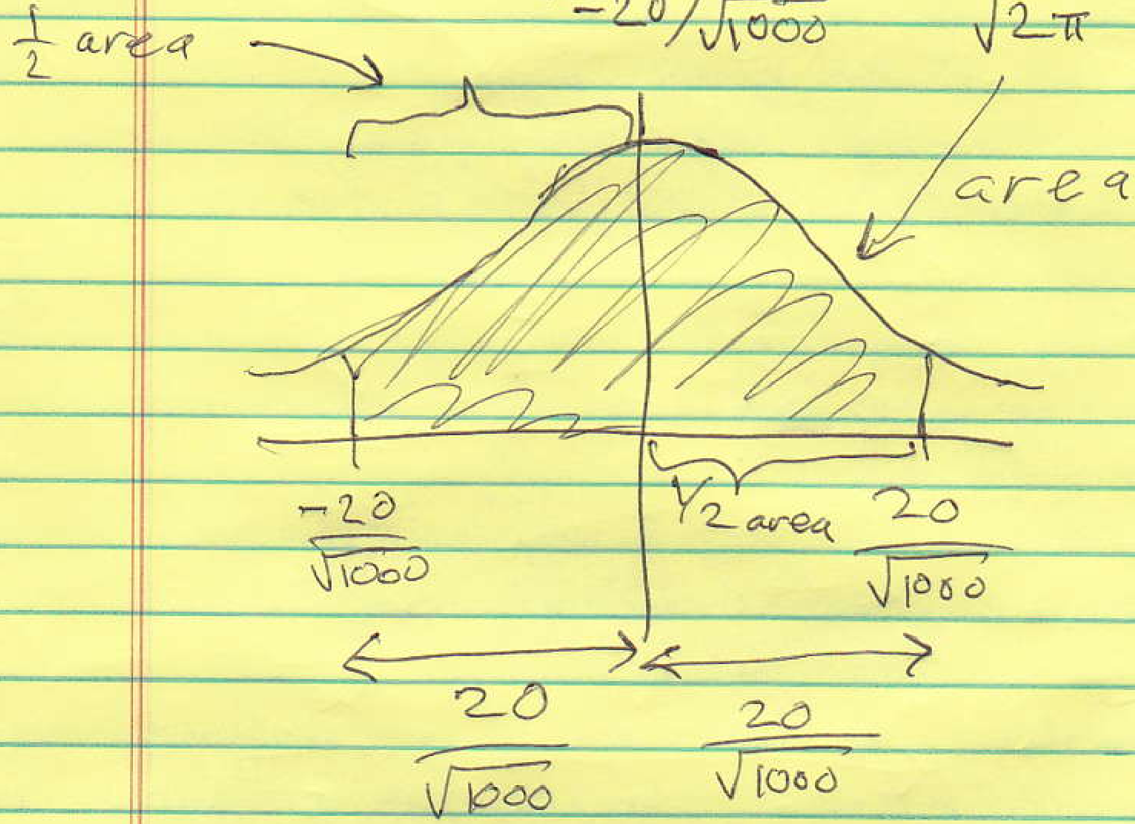
is approximately

$$\int_{a/\sqrt{N}}^{b/\sqrt{N}} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\, dx$$

For the bet, 49% to 51% heads out of $\overset{N}{1000}$ flips means

$$\overbrace{-20}^{a} = 490-510 \leq \#\text{heads} - \#\text{tails} \leq 510-490 = \overbrace{20}^{b}$$

Estimate $\displaystyle\int_{-20/\sqrt{1000}}^{20/\sqrt{1000}} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx$



ULC: Mondays & Wednesdays

Cowan 205 2:30 – 3:30

Review Sessions

Verify the symmetry:

$$f(x) = e^{-x^2/2}/\sqrt{2\pi}$$

$$f(-x) = e^{-(-x)^2/2}/\sqrt{2\pi}$$

$$f(-x) = e^{-x^2/2}/\sqrt{2\pi} = f(x) \checkmark$$

In general, if $\overbrace{f(x) = f(-x)}^{\text{even functions}}$,

then $\int_{-c}^{c} f(x)\,dx = 2\int_{0}^{c} f(x)\,dx$

$$\int_{-20/\sqrt{1000}}^{20/\sqrt{1000}} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx$$

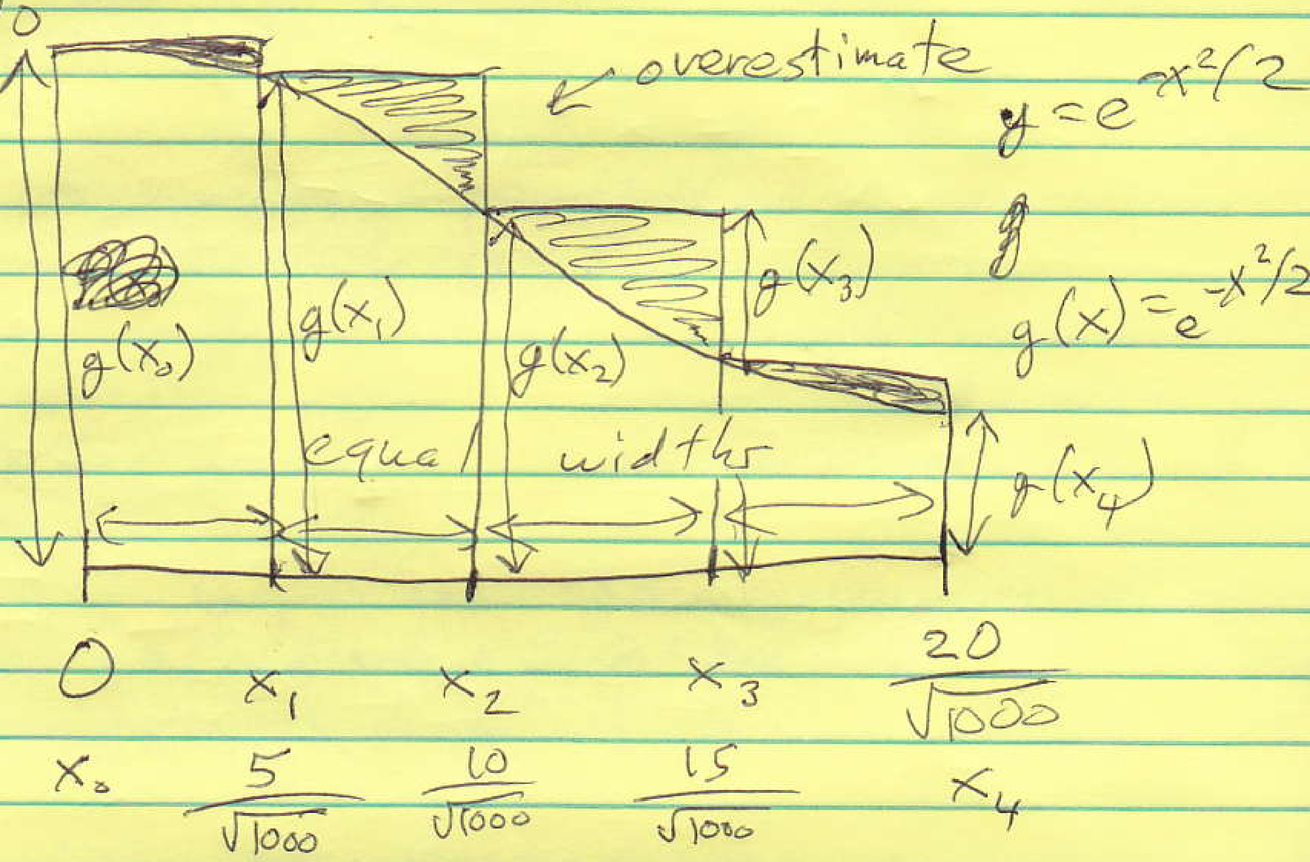
$$= 2\int_{0}^{20/\sqrt{1000}} \overbrace{\left(e^{-x^2/2}/\sqrt{2\pi}\right)}^{\text{integrand}}\,dx$$

Shorter intervals yield more accuracy for numerical estimates of integrals. It's also easier when the integrand is simpler.

$$= \frac{2}{\sqrt{2\pi}} \underbrace{\int_{0}^{20/\sqrt{1000}} e^{-x^2/2}\,dx}$$

Let's estimate this, then multiply by $\frac{2}{\sqrt{2\pi}}$

$$\int_0^{20/\sqrt{1000}} e^{-x^2/2}\,dx$$



left endpoint rule:

left upper corner of rectangle touches curve.

$\boxed{g(x) = e^{-x^2/2}}$

	x_0	$g(x_0)$
x_0	0	1
x_1	0.1581	0.9876
x_2	0.3162	0.9513
x_3	0.4743	0.8936
x_4	0.6324	0.8187

Just need these for left endpoint rule

total area of rectangles:

$$\sum_{\text{rectangles}} (\text{width} \times \text{height})$$

$$= \sum_{k=0}^{3} \underbrace{\frac{5}{\sqrt{1000}}}_{0.1581\ldots} \cdot g(x_k)$$

$$= \Big(g(x_0) \cdot 0.1581 + g(x_1) \cdot 0.1581 + g(x_2) \cdot 0.1581 + g(x_3) \cdot \underbrace{0.1581}_{\substack{\text{width usually} \\ \text{written "}\Delta x\text{"}}} \Big)$$

$$= \big(g(x_0) + g(x_1) + g(x_2) + g(x_3)\big) \cdot 0.1581$$

$$= \left(\sum_{k=0}^{3} g(x_k) \right) \Delta x = \cancel{0.5773}\ \ 0.6059$$

Now multiply by $\frac{2}{\sqrt{2\pi}}$:

~~0.9606~~ 0.4835 about 48%

In general, if $h(x)$ is ↘

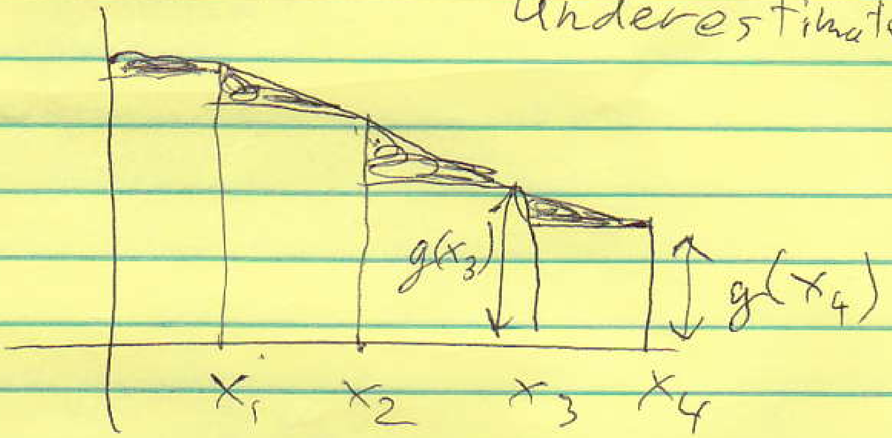
the left endpoint rule

overestimates $\int_0^b h(x)\,dx$

Let's try the right-endpoint

rule:

underestimate



overestimate: $\sum_{k=1}^{4} g(x_k)\,\Delta x = 0.5773$

multiply by $\frac{2}{\sqrt{2\pi}}$ to get: → 0.4606 about 46%

HW:

Estimate probability that between 49% and 51% of 100,000 coin flips will be heads.

Make Δx small enough that overestimate − underestimate is at most 1%.