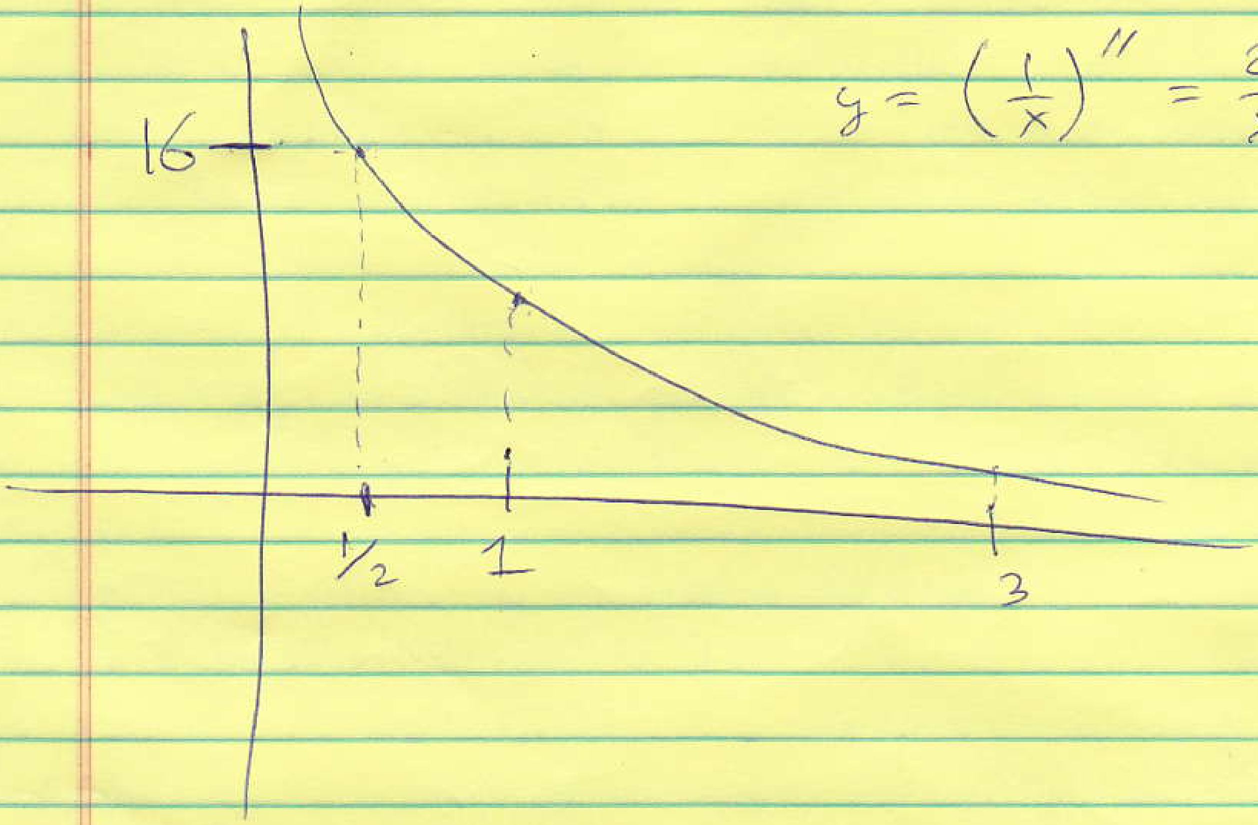
No homework due tomorrow.



$$y = \left(\frac{1}{x}\right)'' = \frac{2}{x^3}$$

$$\int_{1/2}^{1} \frac{1}{x}\,dx$$ with $N = 10$ & Trapezoid Rule

E_T at worst $\pm \dfrac{M(b-a)^3}{12N^2}$

$a = 1/2$
$b = 1$
$N = 10$

where $-M \le \left(\frac{1}{x}\right)'' \le M$

for all x in $[1/2, 1]$

reciprocal

$$\frac{1}{2} \le x \le 1 \Rightarrow 2 \ge \frac{1}{x} \ge 1$$

cube double

$$\Rightarrow 8 \ge \frac{1}{x^3} \ge 1 \Rightarrow 16 \ge \underbrace{\frac{2}{x^3}}_{\left(\frac{1}{x}\right)''} \ge 2 > -16$$

Use $M = 16$

$$E_T \text{ at worst } \pm \frac{16\left(1-\frac{1}{2}\right)^3}{12(10)^2} = \pm 0.001666$$

$$\int_{1/2}^{1} \frac{1}{x}\,dx = \ln|x| \Big|_{1/2}^{1} = \ln 1 - \ln \frac{1}{2}$$

$$= 0 - (-\ln 2) = \ln 2$$

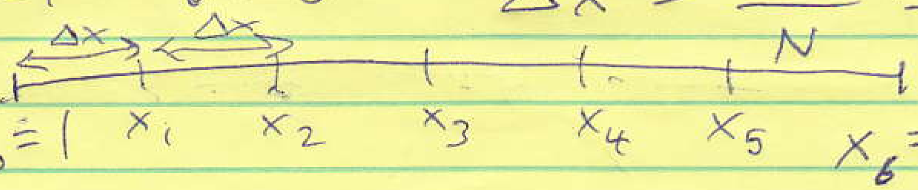
$$\int_1^7 \frac{1}{x}\,dx = \int_{1/7}^{1} \frac{1}{x}\,dx = \ln 7 \qquad \text{etc...}$$

$$\int_1^5 \frac{\sin x}{x}\,dx$$

Estimate this integral with Simpson's Rule and $N = 6$.

$$f(x) = \frac{\sin x}{x}$$

$a = 1 \qquad b = 5 \qquad \Delta x = \frac{b-a}{N} = \frac{4}{6} = \frac{2}{3}$



$$x_k = a + k\Delta x = 1 + \frac{2}{3}k$$

$$\frac{\Delta x}{3}\sum_{k=1}^{N/2}\left(f(x_{2k-2}) + 4f(x_{2k-1}) + f(x_{2k})\right)$$

$$= \frac{2/3}{3}\Bigg[\left(\frac{\sin 1}{1} + 4\,\frac{\sin(5/3)}{5/3} + \frac{\sin(7/3)}{7/3}\right)$$

$$+ \left(\frac{\sin 7/3}{7/3} + 4\,\frac{\sin(9/3)}{9/3} + \frac{\sin 11/3}{11/3}\right)$$

$$+ \left(\frac{\sin 11/3}{11/3} + 4\,\frac{\sin(13/3)}{13/3} + \frac{\sin 5}{5}\right)\Bigg]$$

k	$2k-2$	$2k-1$	$2k$
1	0	1	2
2	2	3	4
3	4	5	6

Getting an error bound...

$$E_S \text{ at worst } \pm \frac{M(b-a)^5}{180N^4}$$

$$a=1 \quad b=5 \quad N=6$$

and $-M \le f^{(4)}(x) \le M$ for all x in $[1, 5]$.

From graphing, $M = 0.3$ works.

$$\underbrace{\left(\frac{\sin x}{x}\right)^{(4)}}_{\text{4th derivative is complicated in this example}} = \left(\frac{4}{x^2} - \frac{24}{x^4}\right)\cos x + \frac{(x^4 - 12x^2 + 24)\sin x}{x^5}$$

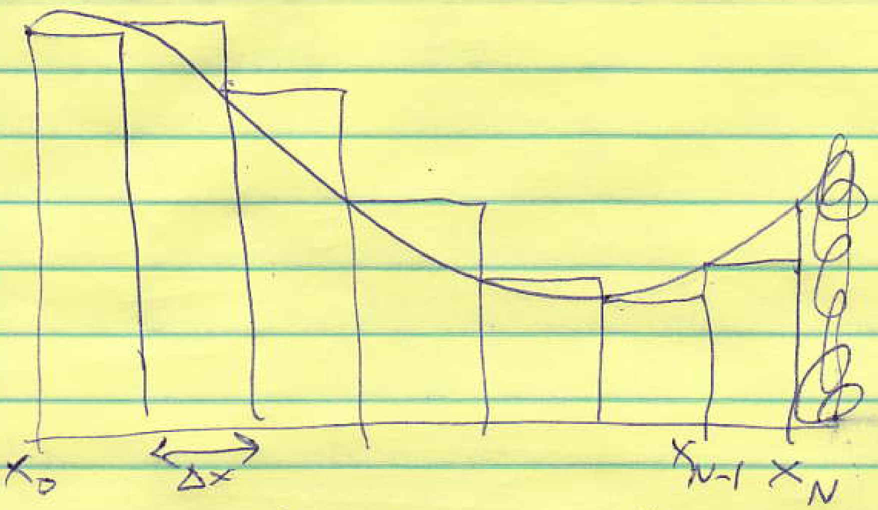
$$E_s \text{ at worst } \pm \frac{0.3\,(5-1)^5}{180 \cdot 6^4}$$

$$= \pm\, 0.001317\ldots$$

$$\int_{A/\sqrt{K}}^{B/\sqrt{K}} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx$$

= probability that $A \le \#\text{heads} - \#\text{tails} \le B$

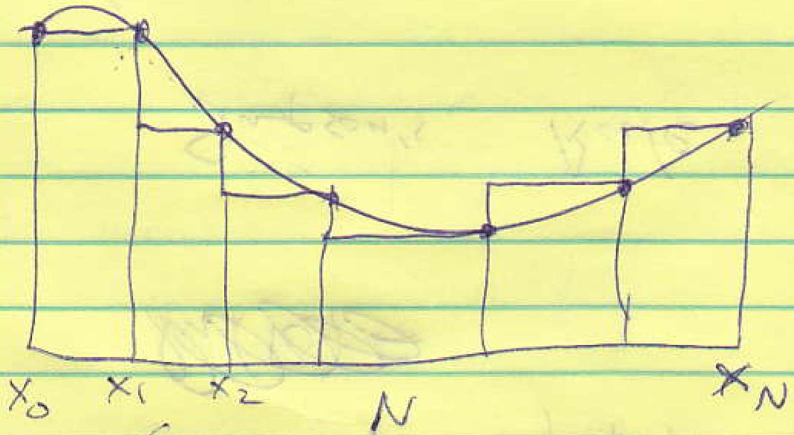
for K coin flips



Left-endpoint rule:

$$\Delta x \sum_{k=0}^{N-1} f(x_k) \qquad \left(= \Delta x \sum_{k=1}^{N} f(x_{k-1}) \right)$$

Right endpoint rule:



$$(\Delta x) \sum_{k=1}^{N} f(x_k)$$