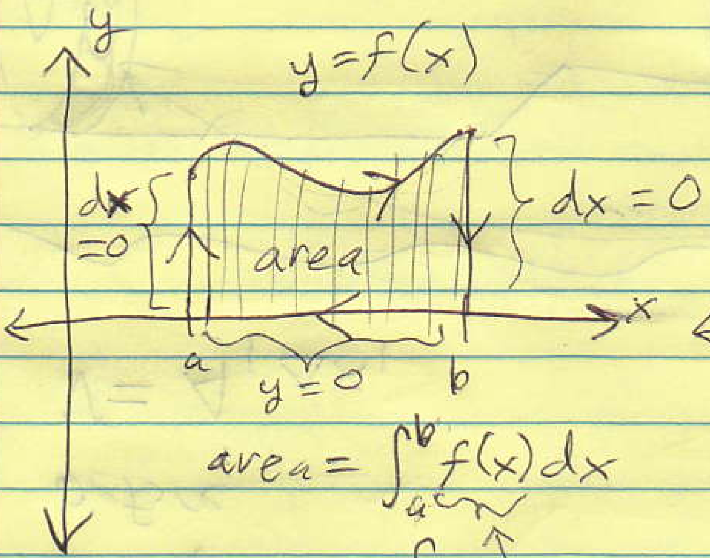
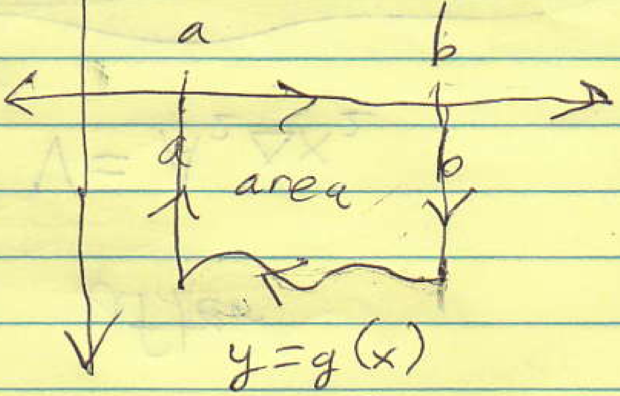
	$dx<0$	$dx=0$	$dx>0$
$dy>0$	↖	↑	↗
$dy=0$	←	•	→
$dy<0$	↙	↓	↘



$$\text{area} = \int_a^b f(x)\,dx$$
$$= \int_{loop} y\,dx$$

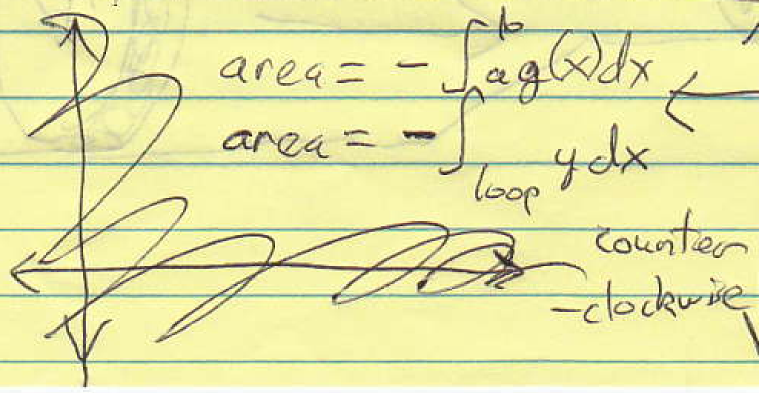
$$\text{area} = -\int_a^b g(x)\,dx$$
$$\text{area} = +\int_b^a g(x)\,dx$$

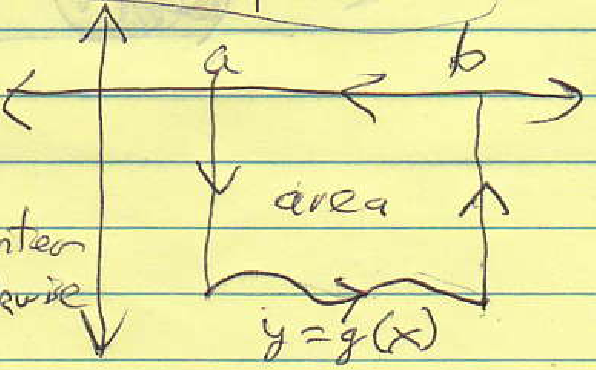


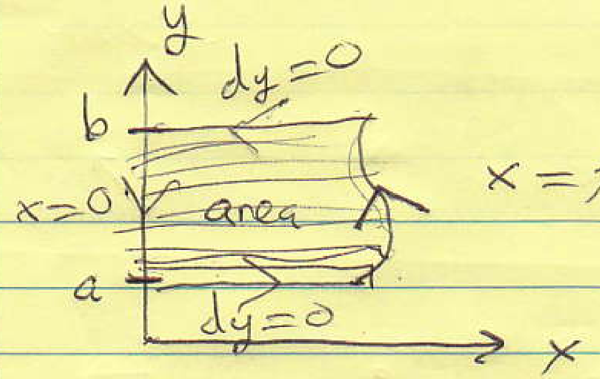
$$\text{area} = \int_{loop} y\,dx$$

clockwise loop $\Rightarrow$ area $= \int_{loop} y\,dx$

$$\text{area} = -\int_a^b g(x)\,dx$$
$$\text{area} = -\int_{loop} y\,dx$$





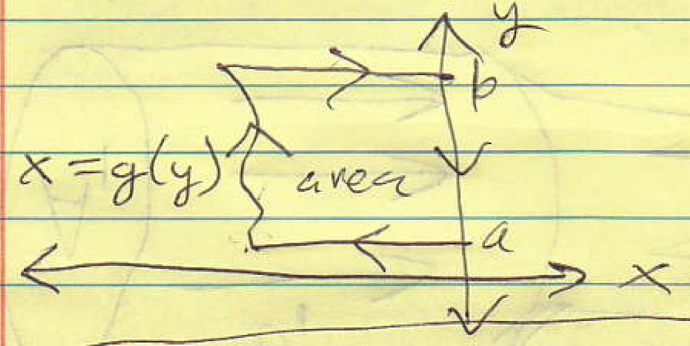


$x=f(y)$

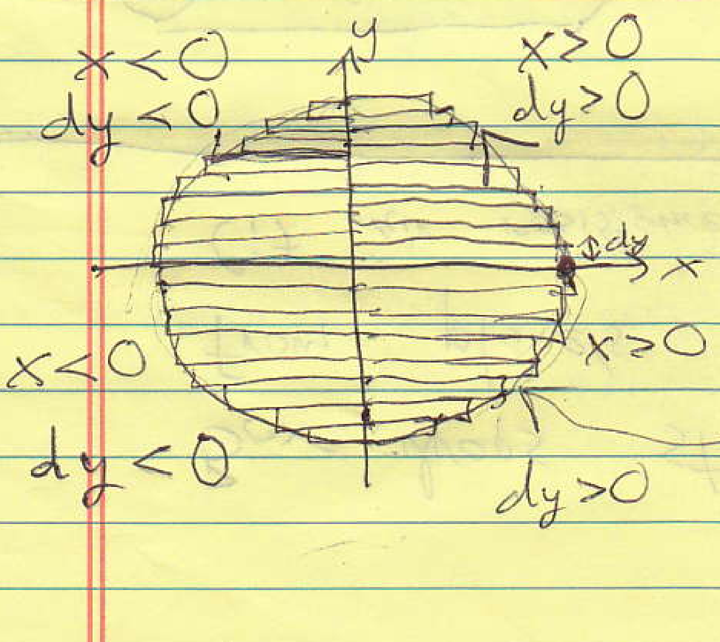
$$\text{area} = \int_a^b f(y)\,dy = \int_a^b x\,dy$$

$$\text{area} = \int_{\text{loop}} x\,dy \quad \text{for counterclockwise loops}$$

$$\text{area} = -\int_{\text{loop}} x\,dy \quad \text{for clockwise loops}$$



$$\text{area} = -\int_a^b g(y)\,dy$$



$x = \cos t$

$y = \sin t$

$0 \le t \le 2\pi$

$\text{area} = \pi$

$$\text{area} = \int_{\text{loop}} x\,dy$$

$$\pi = \text{area} = \int_{\text{loop}} x\, dy = \int_{\text{loop}} \overbrace{\cos t}^{x}\ \overbrace{\cos t\ dt}^{dy}$$

$$dy = (\sin t)'\, dt = \cos t\ dt$$

$$\frac{dy}{dt} = (\sin t)'$$

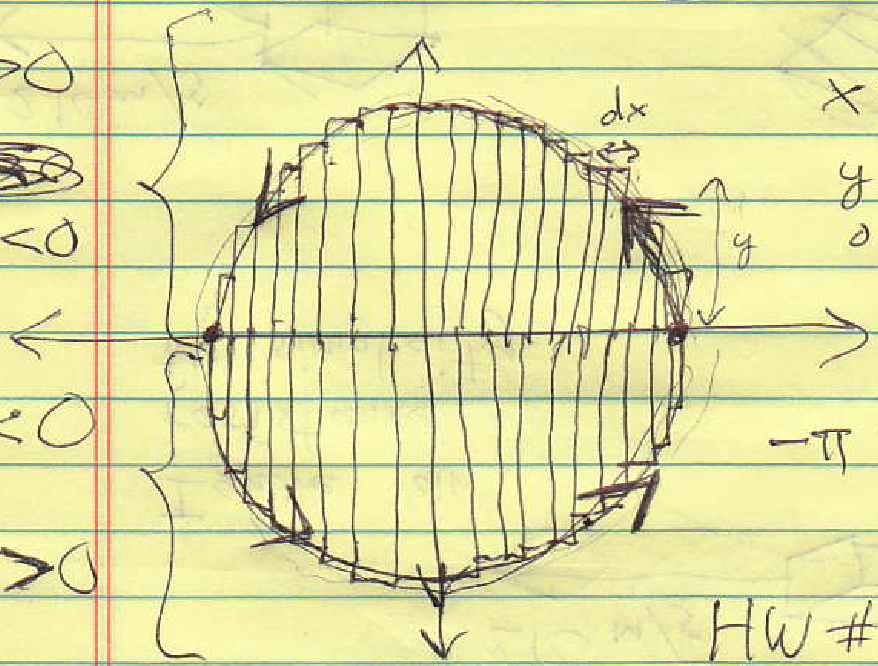
$$\int_0^{2\pi} \cos^2 t\ dt =$$

$y > 0$

$dx < 0$

$y < 0$

$dx > 0$



$x = \cos t$

$y = \sin t$

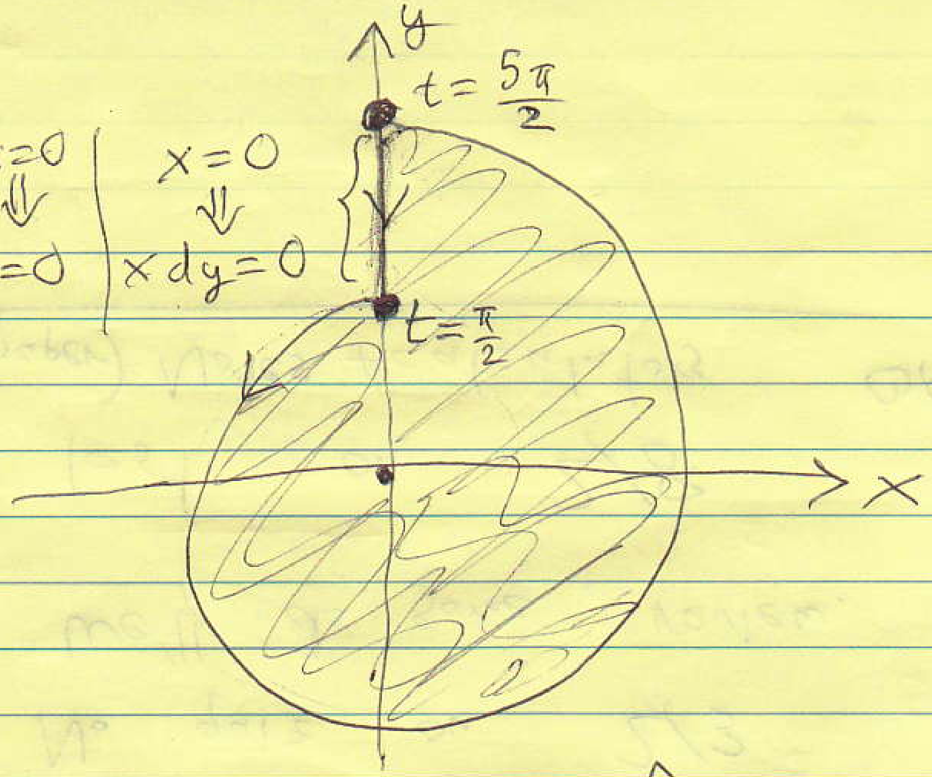
$0 \le t \le 2\pi$

$$-\pi = \int_{\text{loop}} y\, dx$$

HW #1

Express $-\pi = \int_{\text{loop}} y\, dx$ as an integral in terms of t and estimate it with $N = 9$ and the trapezoid rule.

$$dx = 0 \;\Downarrow\; y\,dx = 0 \qquad\Big|\qquad x = 0 \;\Downarrow\; x\,dy = 0$$



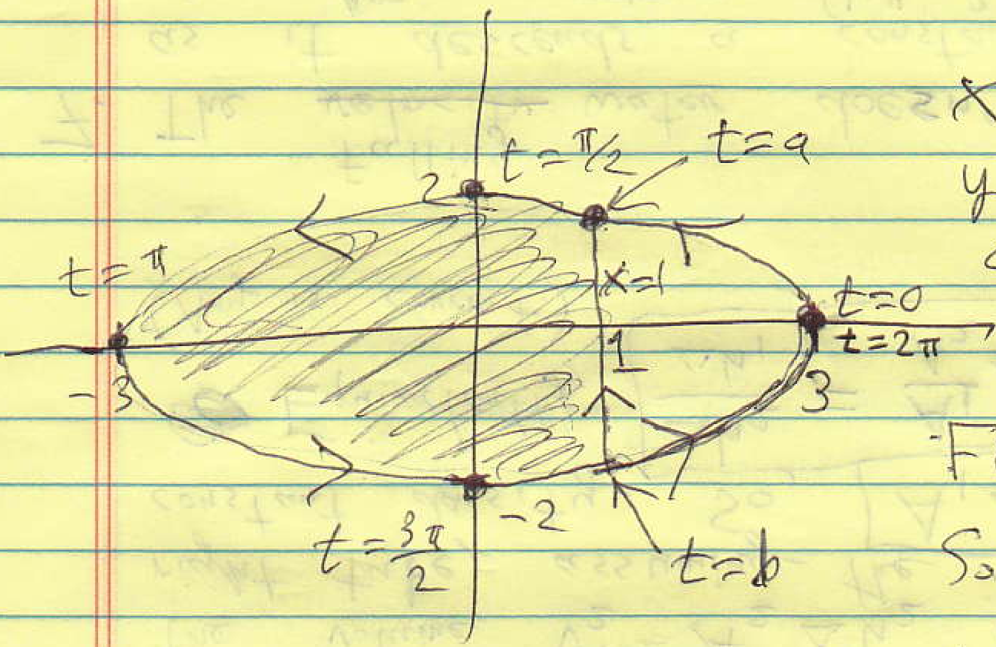
$$x = t\cos t$$
$$y = t\sin t$$
$$\frac{\pi}{2} \le t \le \frac{5\pi}{2}$$

$$\int_{loop} x\,dy \qquad \int_{t=\pi/2}^{t=5\pi/2} x\,dy$$

$$\int_{loop} y\,dx = \int_{t=\pi/2}^{t=5\pi/2} y\,dx$$

HW #2 Express the area above as an integral in terms of t and estimate the area with Simpson's Rule $(N=8)$.

Ellipse with a piece chopped off.



$x = 3\cos t$
$y = 2\sin t$
$0 \le t \le 2\pi$

Find a, b:

Solve $1 = x = 3\cos t$

$b = 2\pi - a$

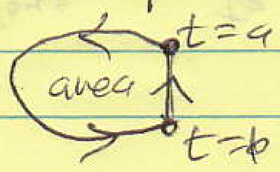
$\frac{1}{3} = \cos t$

$$\cos^{-1}\frac{1}{3} = \underbrace{\arccos}\frac{1}{3} = a$$

always gives an angle in $[0, \pi]$

$dx = 0 \Downarrow y\,dx = 0$ | $\left.\begin{array}{l} x = 1 \\ dy > 0 \end{array}\right\} \Rightarrow x\,dy \neq 0$

$$\int_{\text{loop}} y\,dx = \int_{t=a}^{t=b} y\,dx$$



HW #3 Write area as integral in terms of t.