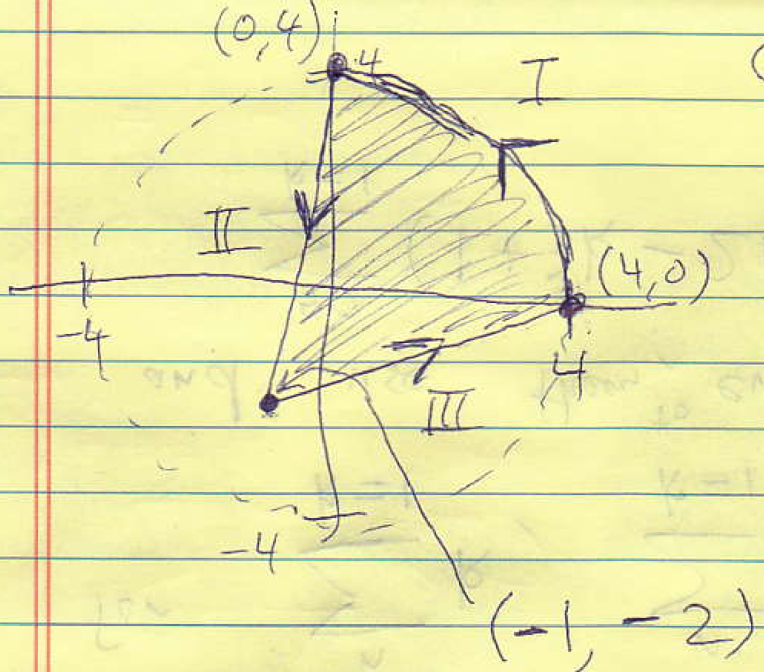
HW #1 Consider a circle with radius 5 and center (0,0). Cut off the left tip at $x=-4$. Express the area as an integral. There should be just 1 variable in your final answer (t, x, y, but just 1).



Circle: radius 4; center (0,0).

area = ?

arc:

$x = 4\cos t$

$y = 4\sin t$

start: $t=0$

end: $t=\pi/2$

counterclockwise loop = I + II + III

$$\text{area} = \int_{\text{loop}} x\,dy = \int_{I} x\,dy + \int_{II} x\,dy + \int_{III} x\,dy$$

I: $dy = (4\sin t)'\,dt = 4\cos t\,dt$

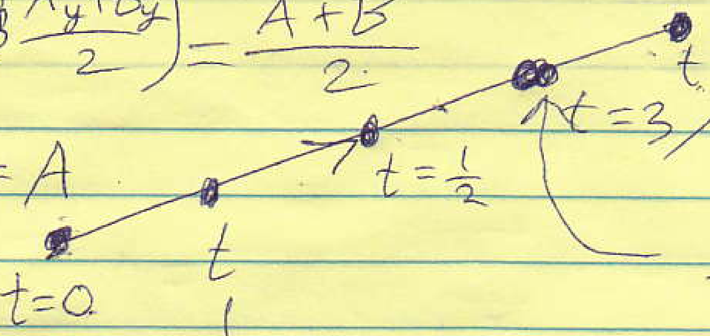
$$\int_I x\,dy = \int_0^{\pi/2} (4\cos t)(4\cos t\,dt) = 4\pi$$

$$\left(\frac{A_x+B_x}{2}, \frac{A_y+B_y}{2}\right)=\frac{A+B}{2}$$

$$(1-1)A+1B=B$$

$$B=(B_x, B_y)$$

$$(A_x, A_y)=A$$



$$\frac{\frac{A+B}{2}+\frac{B}{2}}{2}$$

$$A=1A+0B$$

$$A=(1-0)A+0B$$

$$(1-t)A+tB$$

$$\frac{1}{4}A+\frac{3}{4}B$$

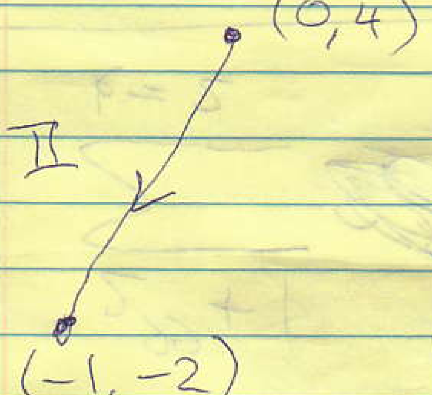
$$x=(1-t)A_x+tB_x$$

$$y=(1-t)A_y+tB_y$$

start: $t=0$

end: $t=1$

II



$$x=(1-t)0+t(-1)=-t$$

$$y=(1-t)4+t(-2)=4-6t$$

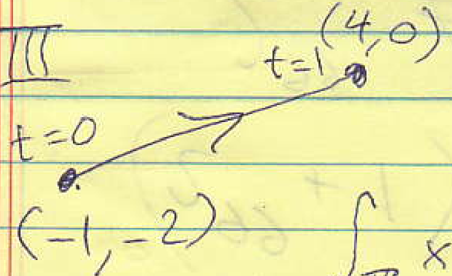
start: $t=0$

end: $t=1$

$$\int_{II} x\,dy=\int_0^1(-t)(-6\,dt)=3$$

$$y=4-6t \Rightarrow dy=-6\,dt$$

III



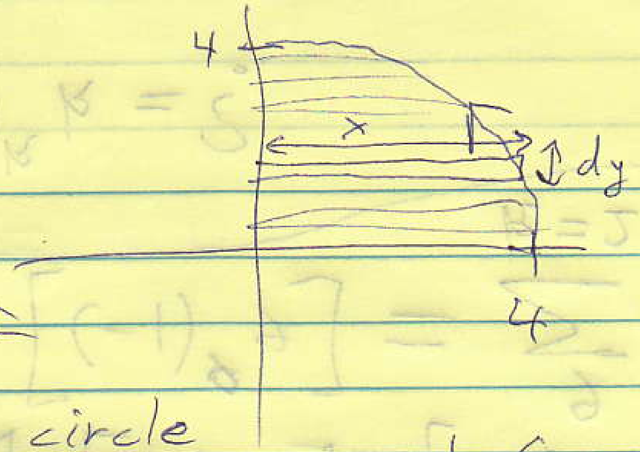
$$x=(1-t)(-1)+t(4)=-1+5t$$

$$y=(1-t)(-2)+t(0)=-2+2t$$

$$\int_{III} x\,dy=\int_0^1(-1+5t)(2\,dt)=3$$

$\int_{I} x\,dy$

area of quarter circle of radius 4 $= \frac{1}{4}(\pi \cdot 4^2) = 4\pi$



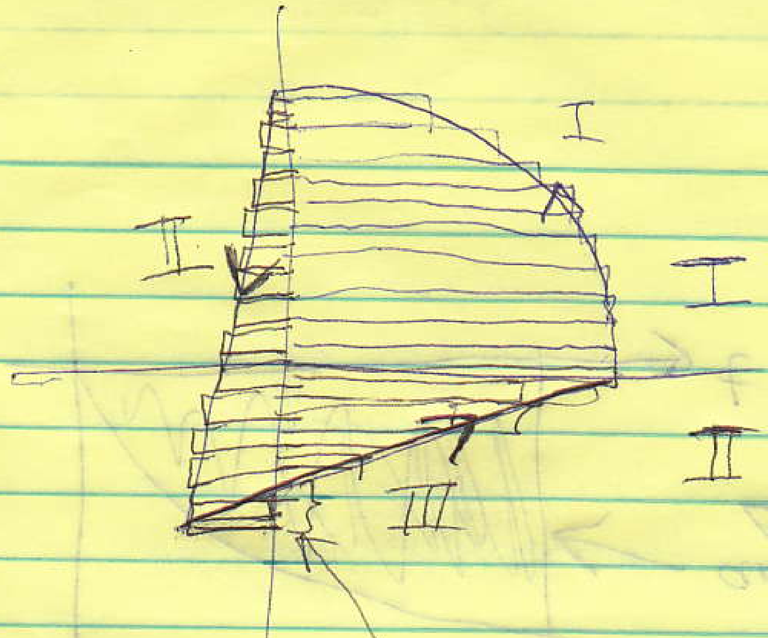
$$\text{area} = \int_{\text{loop}} x\,dy$$

$$= \int_{I} x\,dy + \int_{II} x\,dy + \int_{III} x\,dy$$

$$= 4\pi + 3 + 3$$

$$= 4\pi + 6$$

$$\int_{loop} x\,dy$$



I : $x > 0$, $dy > 0$

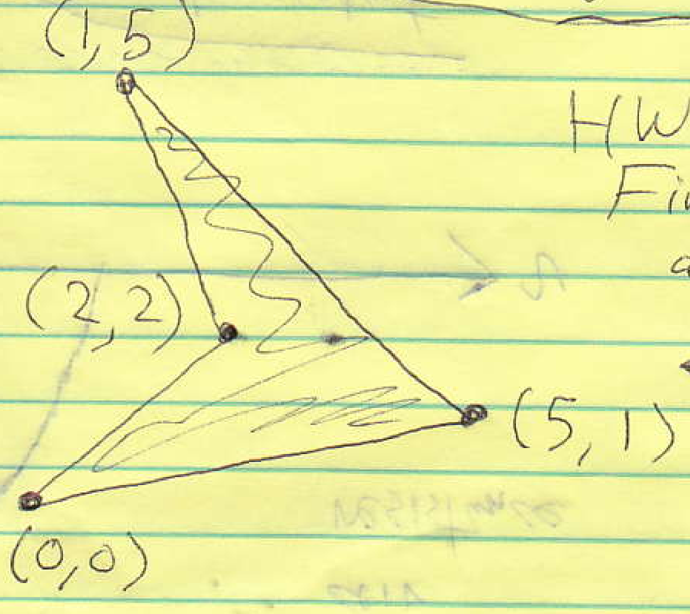
II : $x < 0$, $dy < 0$

First bit of III :

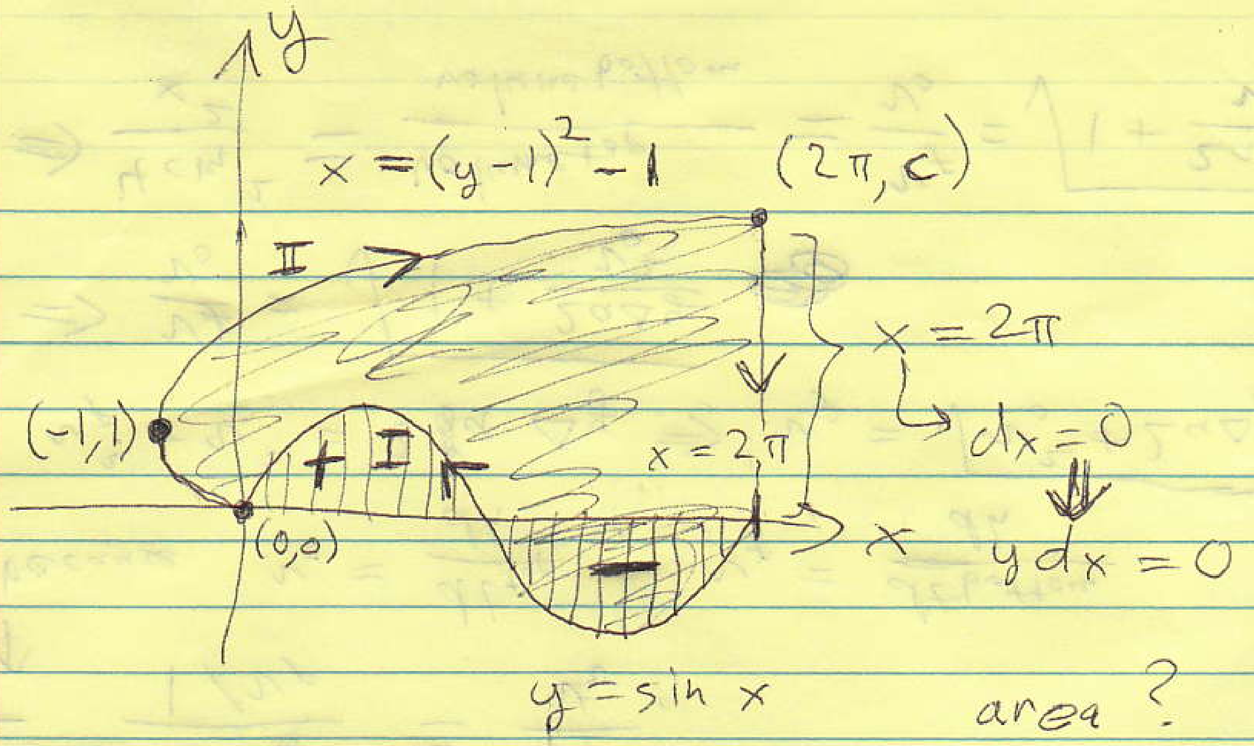
$x < 0$, $dy > 0$

rest of III :

$x > 0$, $dy > 0$



HW #2 : Find the area.



$$\text{area} = \int_{\text{loop}} y\,dx \quad (\text{clockwise})$$

$$= \int_{I} y\,dx + \int_{II} y\,dx$$

$$\int_{I} y\,dx = \int_{2\pi}^{0} \sin x\,dx = -\int_{0}^{2\pi} \sin x\,dx = 0$$

$$\text{area} = \int_{II} y\,dx = ? \qquad x = (y-1)^2 - 1$$

$$dx = 2(y-1)\,dy$$

$$= \int_{0}^{c} y \cdot 2(y-1)\,dy$$

HW #3: Find the area.