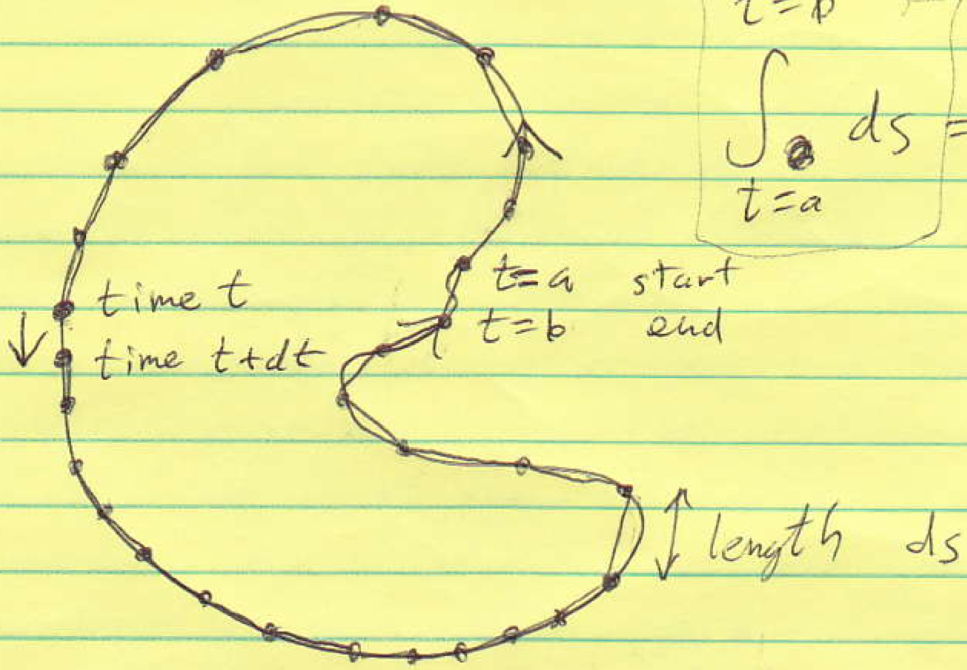
Today: arc-length (continued)

Thursday: test 2

Interpreting dx, dy, ds, dt

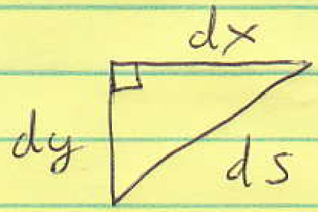
$$\boxed{\int_{t=a}^{t=b} ds} = \text{total arc length}$$



$\frac{dx}{dt}$ = instantaneous x-velocity

$\frac{dy}{dt}$ = instantaneous y-velocity

$\frac{ds}{dt}$ = instantaneous speed

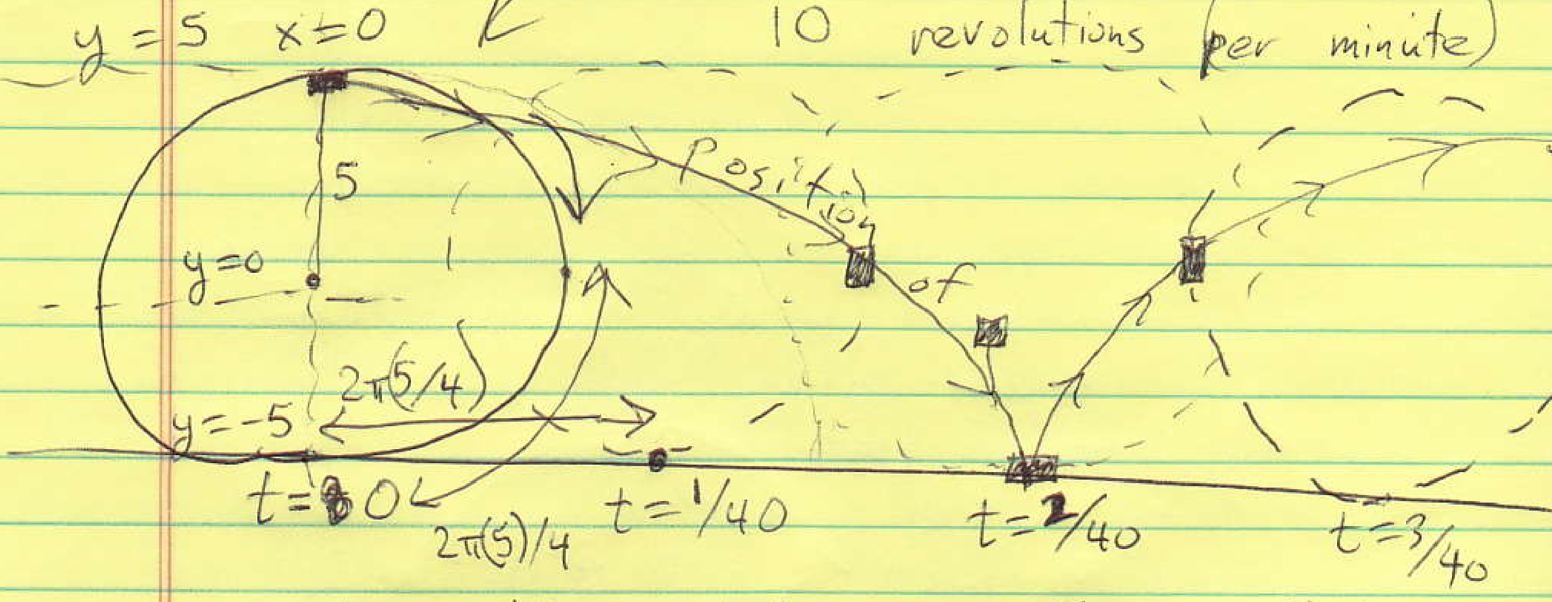


$$ds^2 = dx^2 + dy^2$$

$$\left(\frac{ds}{dt}\right)^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

Wheel with radius 5

10 revolutions (per minute)



$t=1$: 10 revolutions

$t=1/10$: 1 revolution

$t=1/40$: 1/4 rev.

$t=2/40$: 2/4 rev.

$5\cos 0 = 5 \cdot 1 = 5$

$\boxed{y = 5\cos(2\pi t \cdot 10)}$

~~$y = \cos(2\pi t \cdot$ [illegible]$)$~~

$\boxed{x = 5\sin(2\pi t \cdot 10) + 100\pi t}$

$2\pi \times \frac{1}{10} \times 10 = 2\pi$

period of cos()

$$\frac{dx}{dt} = \frac{2\pi \cdot 5/4}{1/40} = 100\pi$$

constant, so $\underbrace{x}_{\substack{x\\ \text{distance}}} = \underbrace{100\pi}_{x\text{-speed}}\underbrace{t}_{\text{time}}$

$$x = 100\pi t + 5\sin(20\pi t)$$
$$y = 5\cos(20\pi t)$$

radius R; $f = \dfrac{\#\text{ revolutions}}{\text{unit time}}$

At top of wheel at $t=0$

$$x = R\sin(2\pi f t) + \underbrace{2\pi R f t}_{\text{rolling to right}}$$

$$y = R\cos(2\pi f t)$$

Time for 1 rev. $= \dfrac{1}{f}$

Time for $\dfrac{1}{4}$ rev. $= \dfrac{1}{4f}$

Homework

Using $R = 40\text{cm}$ & $f = 20 \text{ rev/min}$,

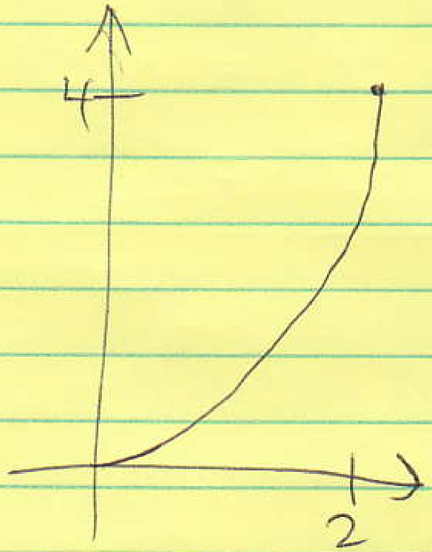
estimate $\displaystyle\int_{t=0}^{t=1/f} ds$

using Trapezoid Rule $(N=8)$.

This is the distance travelled (in cm) if you ride the rolling wheel for one revolution.

Find the length of $y=x^2$

From $x=0$ to $x=2$



$$\int_{x=0}^{x=2} ds = \text{length}$$

$$ds^2 = dx^2 + dy^2$$

$$y=x^2 \Rightarrow dy = 2x\,dx$$

$$ds^2 = dx^2 + (2x\,dx)^2$$

$$ds^2 = dx^2 + 4x^2\,dx^2$$

length

$$\int_0^2 \sqrt{1+4x^2}\,dx \qquad ds^2 = (1+4x^2)\,dx^2$$

$$ds = \sqrt{1+4x^2}\,dx$$

Homework Part 2:

Find the length of the curve

$x = y^3 - y$ from $y=0$ to $y=1$.

Just estimate: Simpson's Rule $(N=6)$.

We can review tomorrow.

Test on Thursday $\left\{ \begin{array}{l} \text{areas} \\ \text{arc lengths} \end{array} \right.$

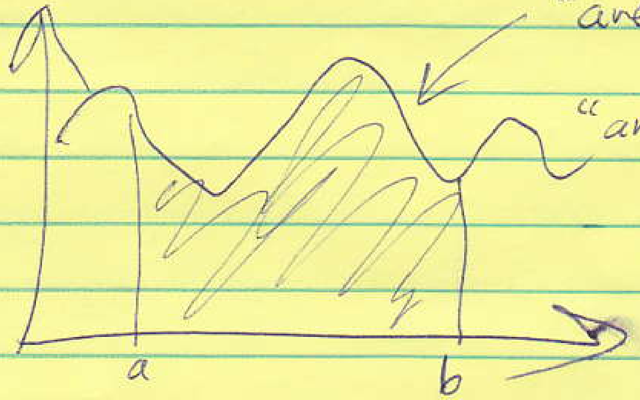
Notes & calculator OK.

$$\int_{t=a}^{t=b} ds = \int_{t=a}^{t=b} \underbrace{\frac{ds}{dt}}_{\text{speed}} \underbrace{dt}_{\text{time}} = \text{distance}$$

speed $= ds/dt$

"area" $= \int$ speed $\times$ (time increment)

"area" = distance



time