- Areas inside loops
- Areas between functions
- Arc length

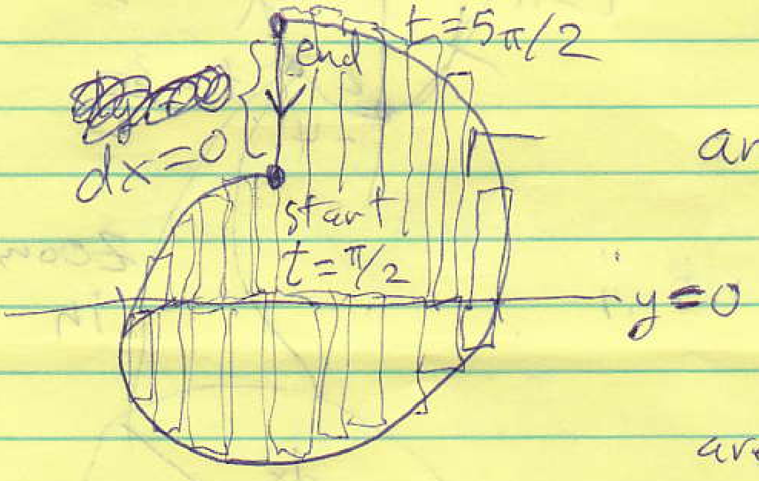
Practice Problems

Area: • See my emailed practice problems

• Solutions: later today

Length: Keisler, section 6.3

Tomorrow's Exam: Notes + calculator OK



$$\text{area} = -\int_{\text{loop}} y\,dx$$

counterclockwise

$$\text{area} = -\int_{t=\pi/2}^{t=5\pi/2} y\,dx$$

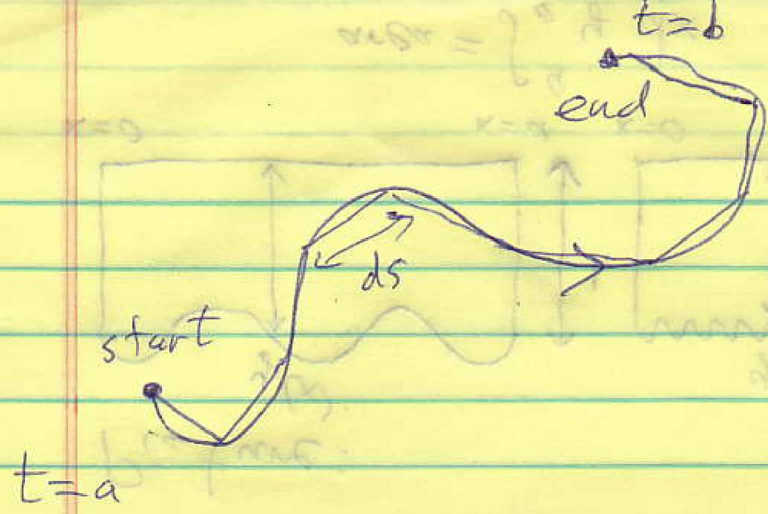
$x = t\cos t$

$y = t\sin t$

$$dx = (t\cos t)'\,dt$$

$$dx = (\cos t + t(-\sin t))\,dt$$

$$\text{area} = -\int_{\pi/2}^{5\pi/2} (t\sin t)(\cos t + t(-\sin t))\,dt$$



$x = f(t)$

$y = g(t)$

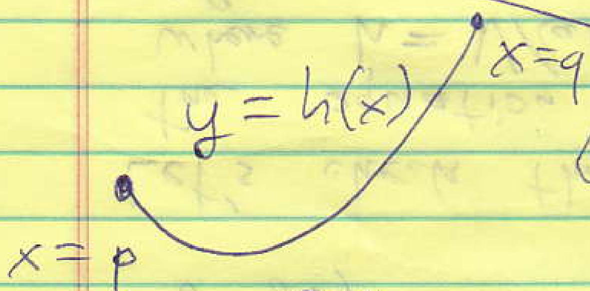
$a \le t \le b$

$$\text{length} = \int_{t=a}^{t=b} ds \qquad \boxed{ds = \sqrt{dx^2 + dy^2}}$$

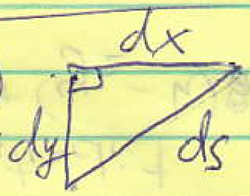
$$dx = f'(t)\,dt$$

$$dy = g'(t)\,dt$$

$$ds = \boxed{\underbrace{\sqrt{f'(t)^2 + g'(t)^2}}_{\frac{ds}{dt} = \text{speed}}\,dt}$$



$ds = \sqrt{dx^2 + dy^2}$

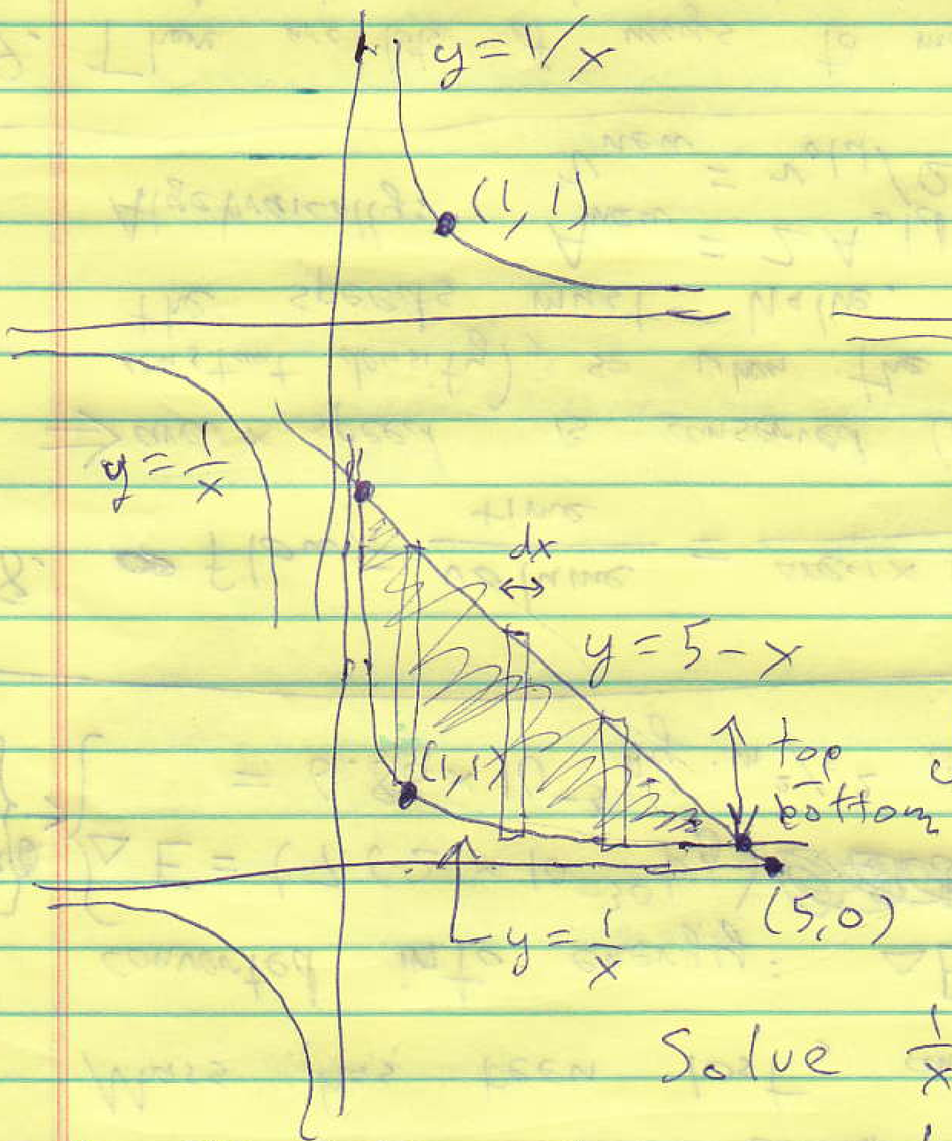


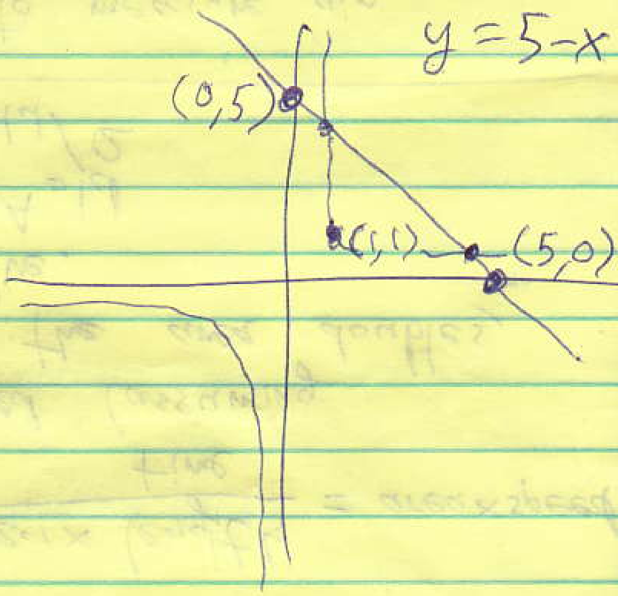
$$dy = h'(x)\,dx$$

$$\text{length} = \int_{x=p}^{x=q} ds \qquad ds = \sqrt{dx^2 + h'(x)^2\,dx^2}$$

$$ds = \sqrt{1 + h'(x)^2}\,dx$$

Find area between $y = \frac{1}{x}$ and $y = 5 - x$, with x restricted to be between the 2 ~~positive x-coord~~ crossing points.





$$\int_a^b (\underbrace{\text{top}}_{5-x} - \underbrace{\text{bottom}}_{1/x})\,dx$$

Solve $\frac{1}{x} = 5 - x$

$$1 = (5 - x)x$$

$$1 = 5x - x^2$$

$$x^2 - 5x + 1 = 0$$

$$x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 1 = 0$$

$$(x - 5/2)^2 - (5/2)^2 + 1 = 0$$

$$\left(x - \frac{5}{2}\right)^2 = \underbrace{\left(\frac{5}{2}\right)^2 - 1}_{21/4}$$

$$x - 5/2 = \pm\sqrt{21/4}$$

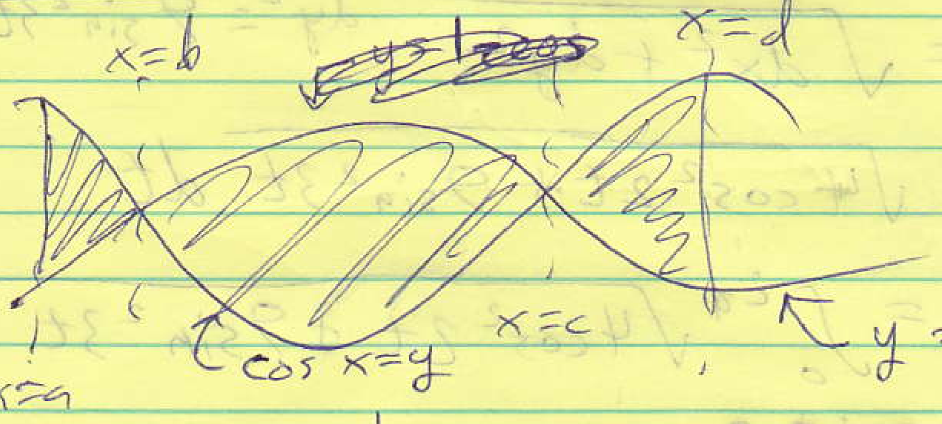
$$x = \frac{5}{2} \pm \frac{\sqrt{21}}{2}$$

$$\text{area} = \int_{5/2-\frac{\sqrt{21}}{2}}^{5/2+\frac{\sqrt{21}}{2}} \left(5 - x - \frac{1}{x}\right) dx$$

$$= \left(5x - \frac{x^2}{2} - \ln|x|\right)\Bigg|_{5/2-\sqrt{21}/2}^{5/2+\sqrt{21}/2}$$

$$= 5\left(\frac{5+\sqrt{21}}{2}\right) - \frac{((5+\sqrt{21})/2)^2}{2} - \ln\frac{5+\sqrt{21}}{2}$$
$$- \left[5\left(\frac{5-\sqrt{21}}{2}\right) - \frac{((5-\sqrt{21})/2)^2}{2} - \ln\frac{5-\sqrt{21}}{2}\right]$$

$$\approx 8.32$$

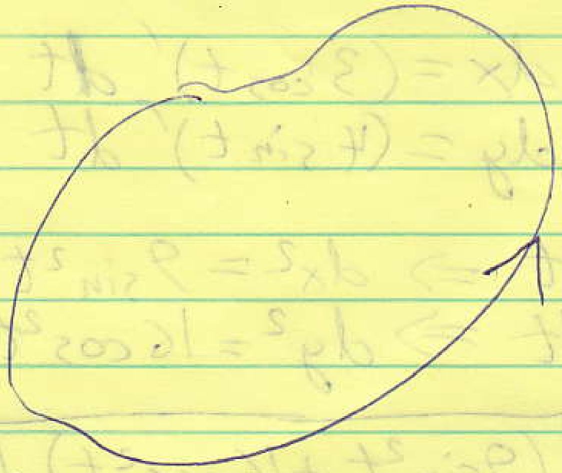


$$\text{area} = \int_a^b (\text{top}-\text{bottom})\,dx + \int_b^c (\text{top}-\text{bottom})\,dx + \int_c^d (\text{top}-\text{bottom})\,dx$$

$a = \pi/4$

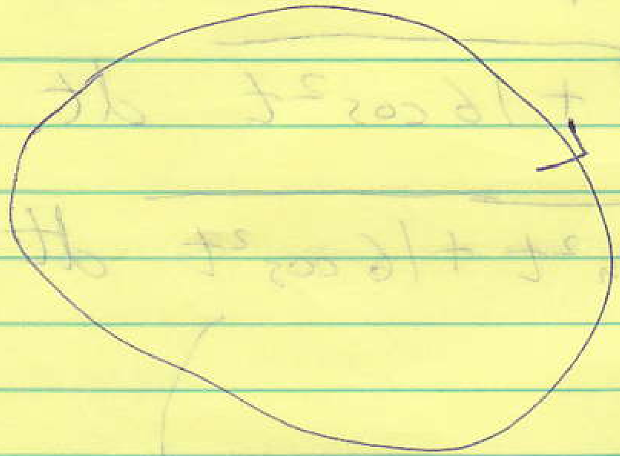
$d = 2\pi$

Estimate b, c



$$\text{area} = \int_{\text{loop}} x\,dy$$

$$= -\int_{\text{loop}} y\,dx$$



$$\text{area} = \int_{\text{loop}} y\,dx$$

$$= -\int_{\text{loop}} x\,dy$$

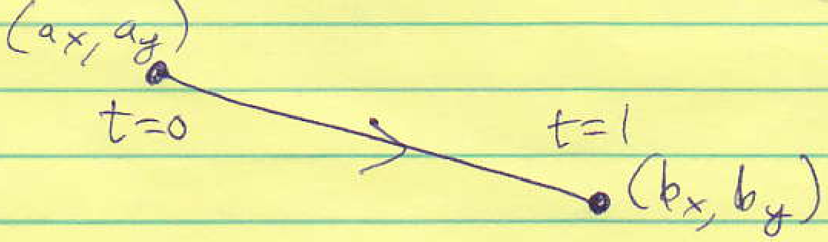
$$x = f(t) \Rightarrow dx = f'(t)\,dt$$

$$x = g(y) \Rightarrow dx = g'(y)\,dy$$

$$y = f(x) \Rightarrow dy = f'(x)\,dx$$

$$y = h(t) \Rightarrow dy = h'(t)\,dt$$

Line segment parametrization:



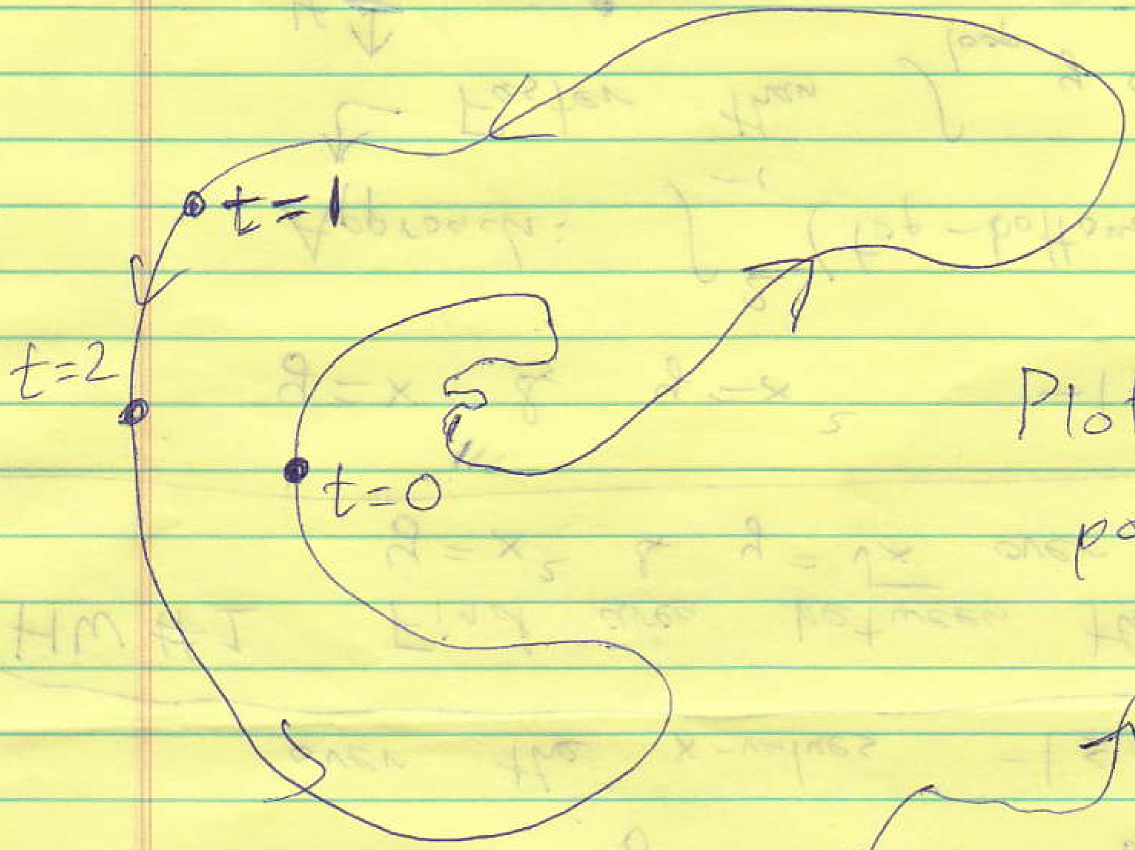
$$x = (1-t)a_x + t\,b_x$$

$$y = (1-t)a_y + t\,b_y$$

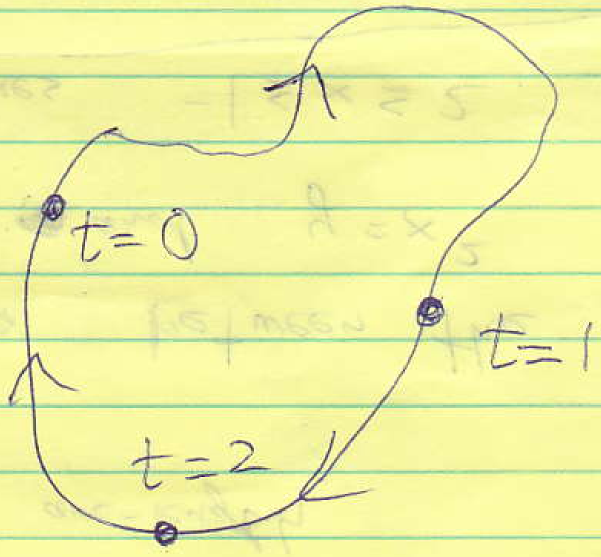
Direction of curves

$x = f(t) \quad y = g(t)$

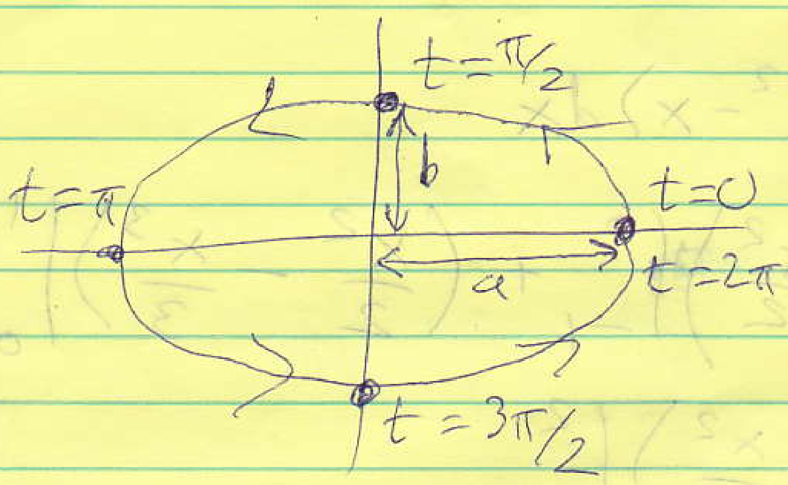
$0 \le t \le 5$



Plot 3 points

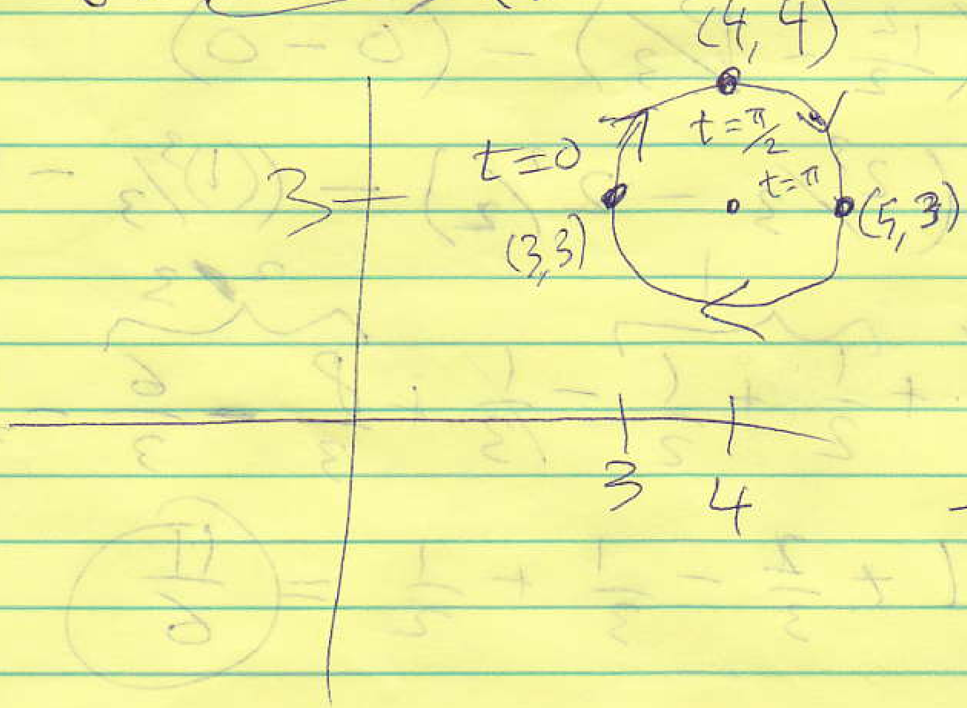


Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$x = a \cos t$
$y = b \sin t$

~~$x = \cos(\sqrt{t}/2\pi)$~~
~~$y = 3 - \sin($~~



$x = 4 - \cos t$
$y = 3 + \sin t$

$0 \le t \le 2\pi$

$t = 0$:

$x = 4 - 1 = 3$
$y = 3 + 0 = 3$

$t = \pi/2$:

$x = 4 - 0 = 4$
$y = 3 + 1 = 4$

$t = \pi$

$x = 4 + 1 = 5$
$y = 3 + 0 = 3$