6.1

Place the pyramid on its side with the apex at $x = 0$ and the base at $x = h$. We use the fact that at any point x between 0 and h, the cross section has area proportional to x^2, so that

$$\frac{A(x)}{x^2} = \frac{B}{h^2},$$

$$A(x) = \frac{Bx^2}{h^2}.$$

The volume is then

$$V = \int_0^h \frac{Bx^2}{h^2}\,dx = \frac{1}{3}\cdot\frac{Bx^3}{h^2}\Big]_0^h = \frac{1}{3}\cdot\frac{Bh^3}{h^2} = \frac{1}{3}Bh.$$

The solution is $V = (\frac{1}{3})Bh$.

EXAMPLE 2 A wedge is cut from a cylindrical tree trunk of radius 3 ft, by cutting the tree with two planes meeting on a line through the axis of the cylinder. The wedge is 1 ft thick at its thickest point. Find its volume.

Section 6.1

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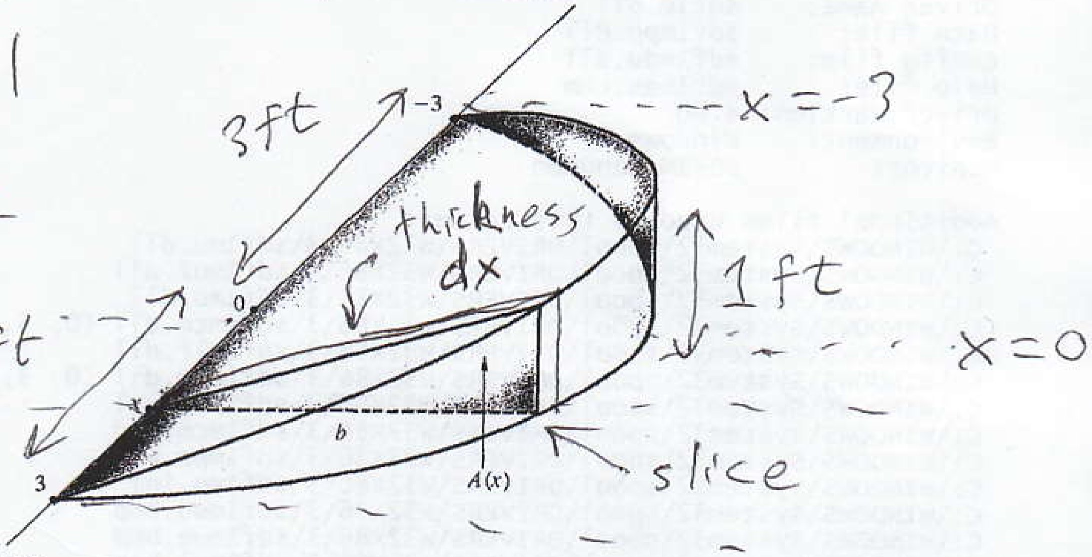


Figure 6.1.8 Example 2

The wedge is shown in Figure 6.1.8. The cross sections perpendicular to the x-axis are similar triangles. Place the edge along the x-axis with x from -3 to 3. At the thickest point, where $x = 0$, the cross section is a triangle with base 3 ft and altitude 1 ft. The base of the cross section triangle at x is

$$b = \sqrt{9 - x^2},$$

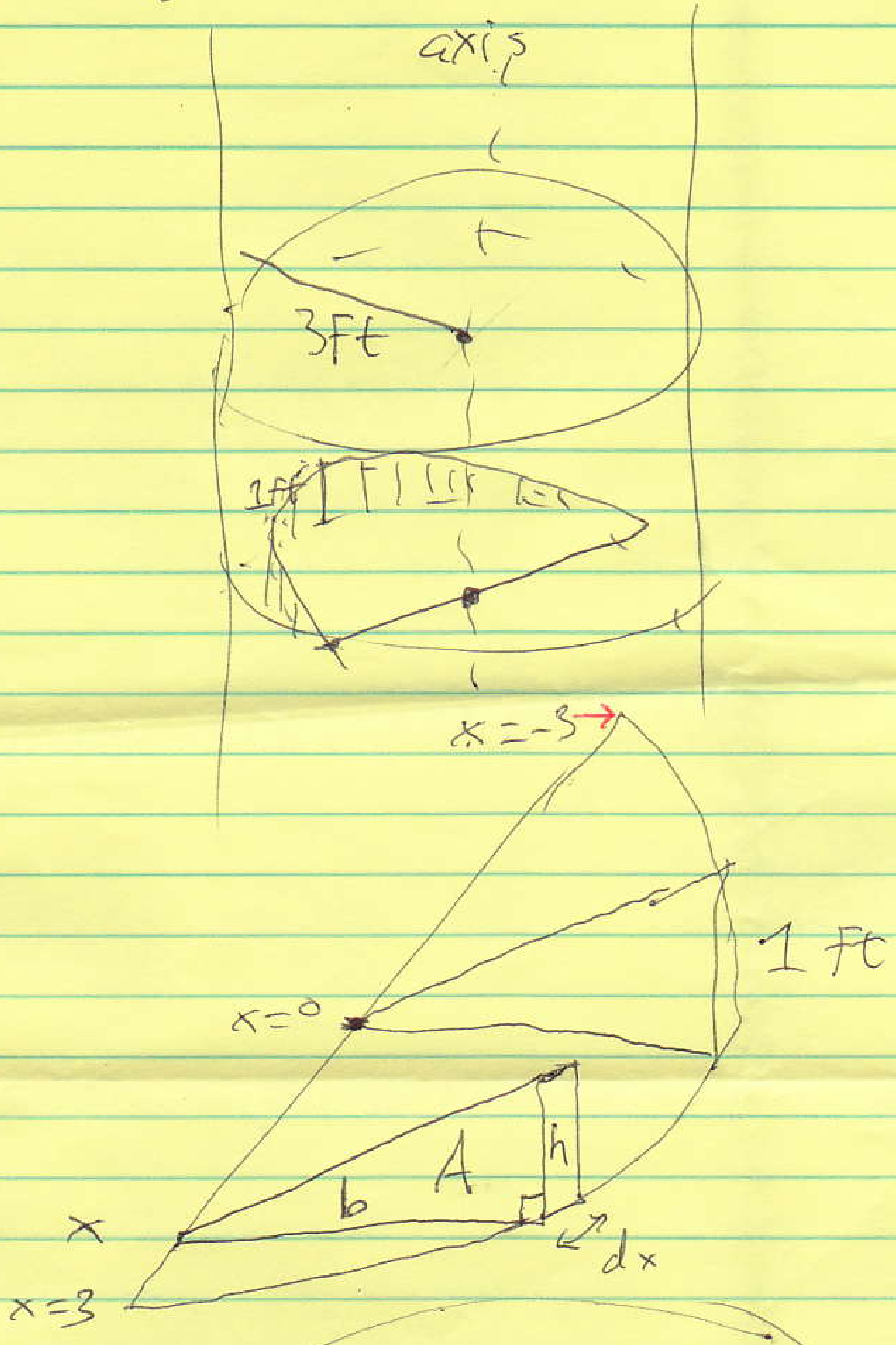
and the altitude is

$$\tfrac{1}{3}b = \tfrac{1}{3}\sqrt{9 - x^2}.$$

The area of the cross section is

$$A(x) = \tfrac{1}{2}\cdot\text{base}\cdot\text{altitude} = \tfrac{1}{2}b\cdot\tfrac{1}{3}b = \tfrac{1}{6}b^2 = \tfrac{1}{6}(9 - x^2).$$

Today: volume by slices

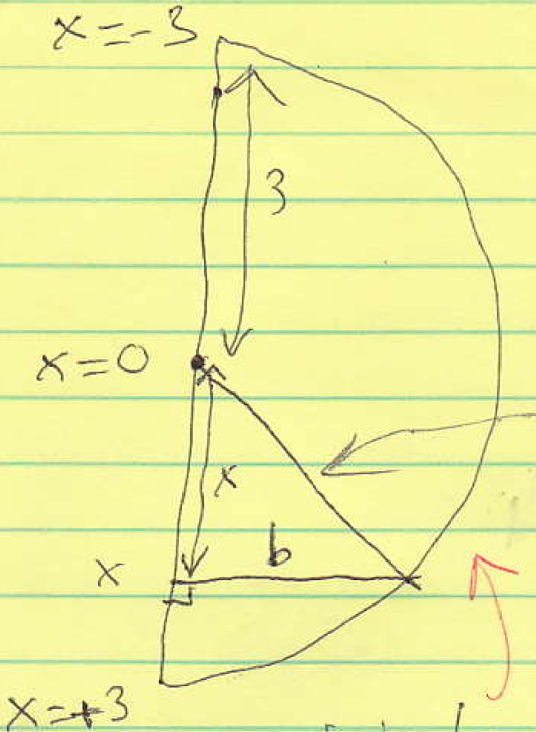


volume of slice

$= A\,dx$

$A = ?$ $A = \frac{1}{2} b h$

$b = ?$ $h = ?$



$\sqrt{x^2+b^2} = \underset{r}{3}$

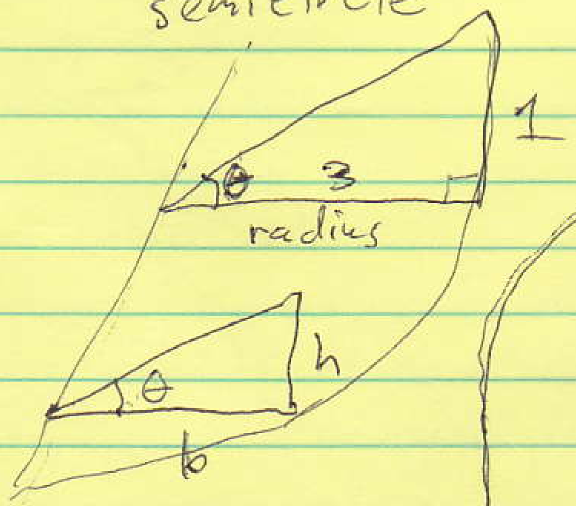
It's a radius

semicircle

$x^2+b^2=9$

$b^2 = 9-x^2$

$b = \sqrt{9-x^2}$



$\tan\theta = \underbrace{\frac{1}{3} = \frac{h}{b}}$

$b = 3h$

$h = \frac{1}{3}\sqrt{9-x^2} \leftarrow b/3 = h$

Volume of slice $= A\,dx = \frac{1}{2} b h \, dx$

$= \frac{1}{2}\sqrt{9-x^2}\,\frac{1}{3}\sqrt{9-x^2}\,dx$

$= \frac{1}{6}(9-x^2)\,dx$

$$V = \int_{-3}^{3} \frac{1}{6}\, \overbrace{(9-\underbrace{x^2}_{ft})}^{ft^2}\, \overbrace{dx}^{ft}$$

$$= \frac{1}{6}\left(9x - \frac{x^3}{3}\right)\Big|_{-3}^{3}$$

$$= \frac{1}{6}\left(9\cdot 3 - \frac{27}{3}\right) - \frac{1}{6}\left(9(-3) - \frac{-27}{3}\right)$$

$$= \frac{1}{6}(27-9) + \frac{1}{6}(27-9)$$

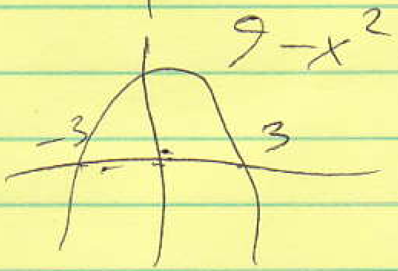
$$= \frac{1}{6}(18) + \frac{1}{6}(18) = 3+3 = \boxed{6\ ft^3}$$

$$9-x^2 = 9-(-x)^2$$

Alternative method

$9-x^2$ is even

$$\int_{-3}^{3} (9-x^2)\,dx$$



$$= 2\int_0^3 (9-x^2)\,dx$$

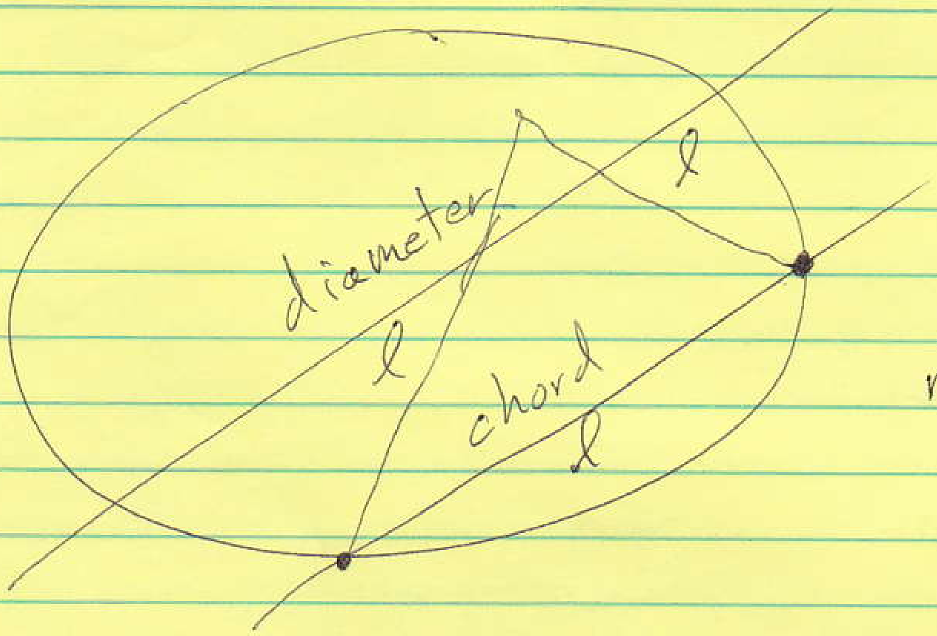
$$V = 2\int_0^3 \frac{1}{6}(9-x^2)\,dx =$$

$$= (2/6)(9x - x^3/3)\Big|_0^3 = \frac{1}{3}(9\cdot 3 - 27/3)$$

$$= \frac{1}{3}(27-9) = \frac{1}{3}(18) = 6\ ft^3$$

HW #1

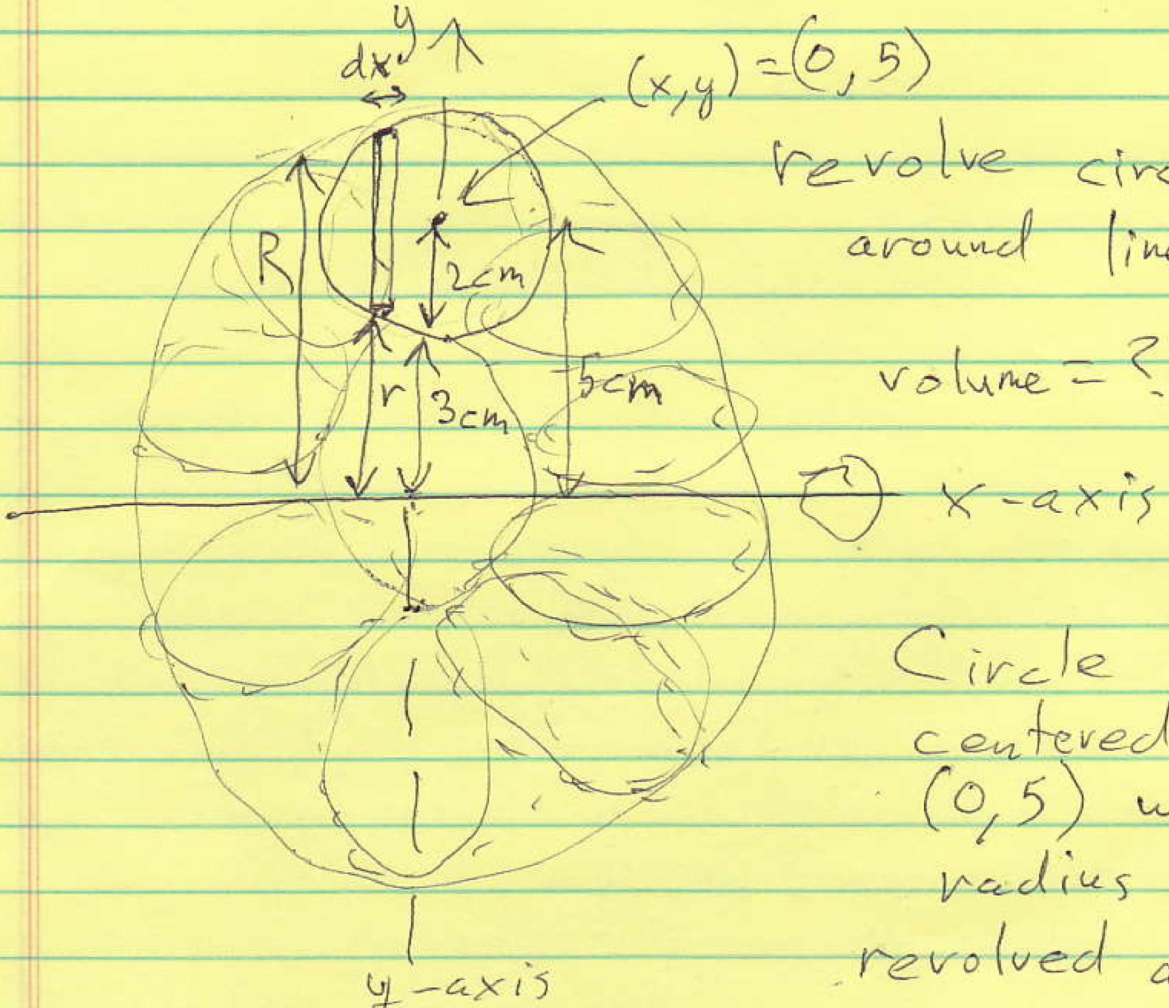
- Circular base of radius 5 in
- ~~Pa~~ Pick a diameter



radius 5 in

- For each chord parallel to the diameter, make that chord the base of an equilateral triangle pointing "out of the paper"
- Find the total volume (in in^3).

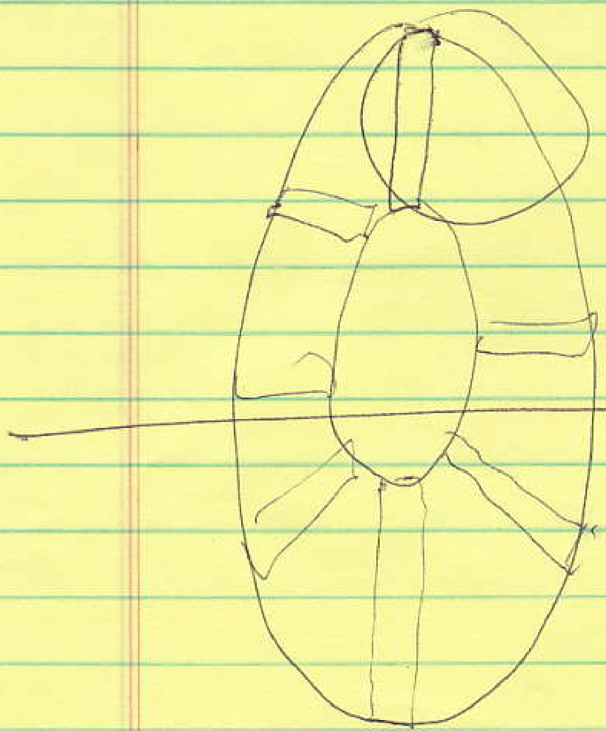
Volume of a donut

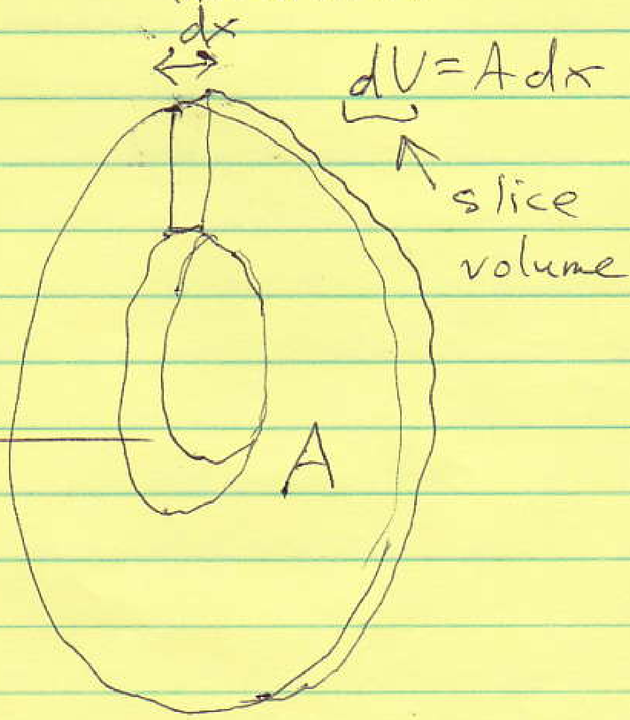


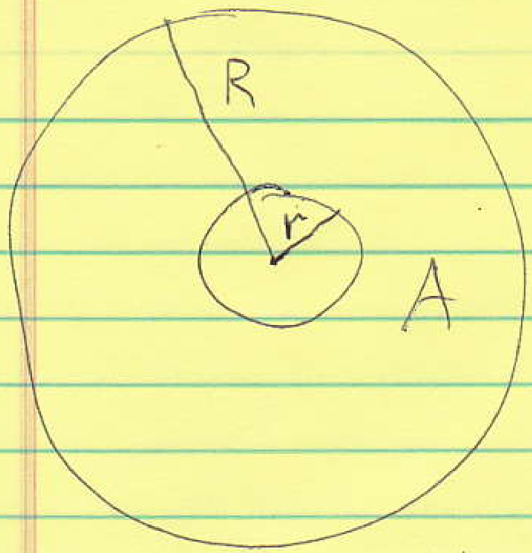
revolve circle around line

volume = ?

Circle centered at (0,5) with radius 2, revolved around x-axis.

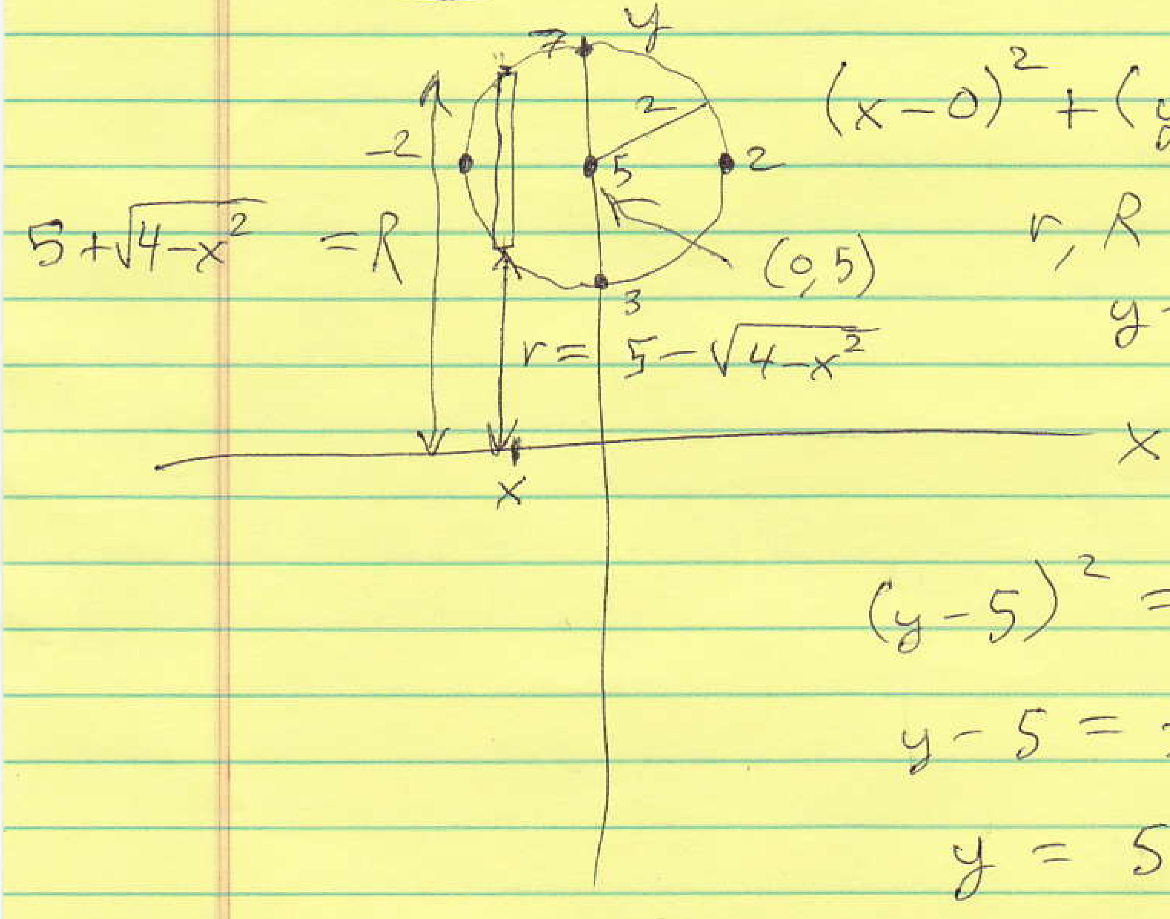






Two concentric circles

$$A = \pi R^2 - \pi r^2$$
$$= \pi(R^2 - r^2)$$



$$(x-0)^2 + (y-5)^2 = 2^2$$

r, R are y-coordinates

$$(y-5)^2 = 2^2 - x^2$$
$$y - 5 = \pm\sqrt{4-x^2}$$
$$y = 5 \pm \sqrt{4-x^2}$$

$$dV = A\,dx = \pi(R^2 - r^2)\,dx$$
$$dV = \pi\left(\left(5+\sqrt{4-x^2}\right)^2 - \left(5-\sqrt{4-x^2}\right)^2\right)dx$$
$$V = \int_{x=-2}^{x=2} dV$$

$$V = \int_{-2}^{2} \pi \left(\left(5 + \sqrt{4-x^2}\right)^2 - \left(5 - \sqrt{4-x^2}\right)^2 \right) dx$$

Simplify ---

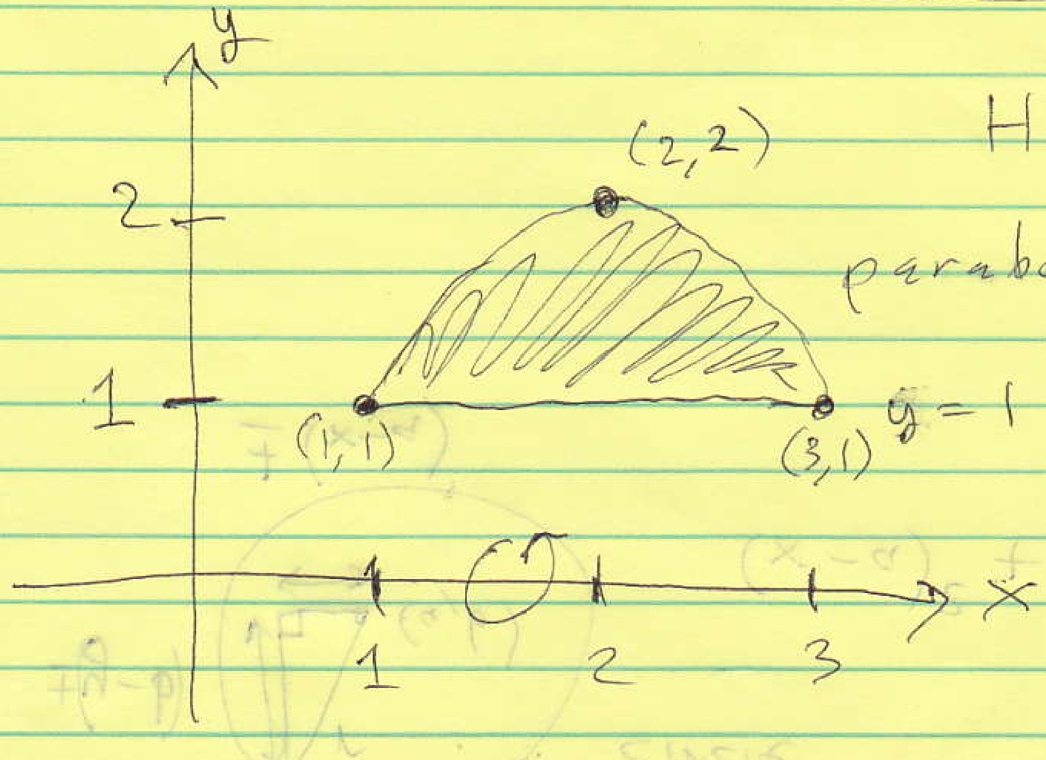
$$V = 2\pi \int_0^2 20\sqrt{4-x^2}\, dx$$

$\sqrt{4-x^2}$ is even

$$(a+b)^2 - (a-b)^2$$

$$= (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$$

$$= 4ab$$

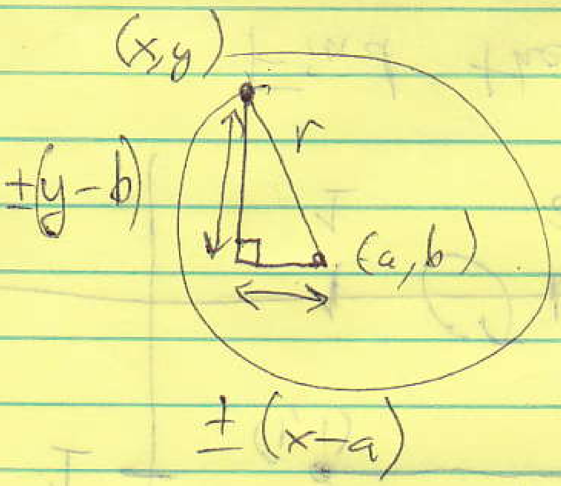


HW #2

parabola $y = 2-(x-2)^2$

Revolve region around x-axis

Find the exact volume.



circle

$$(x-a)^2+(y-b)^2=r^2$$