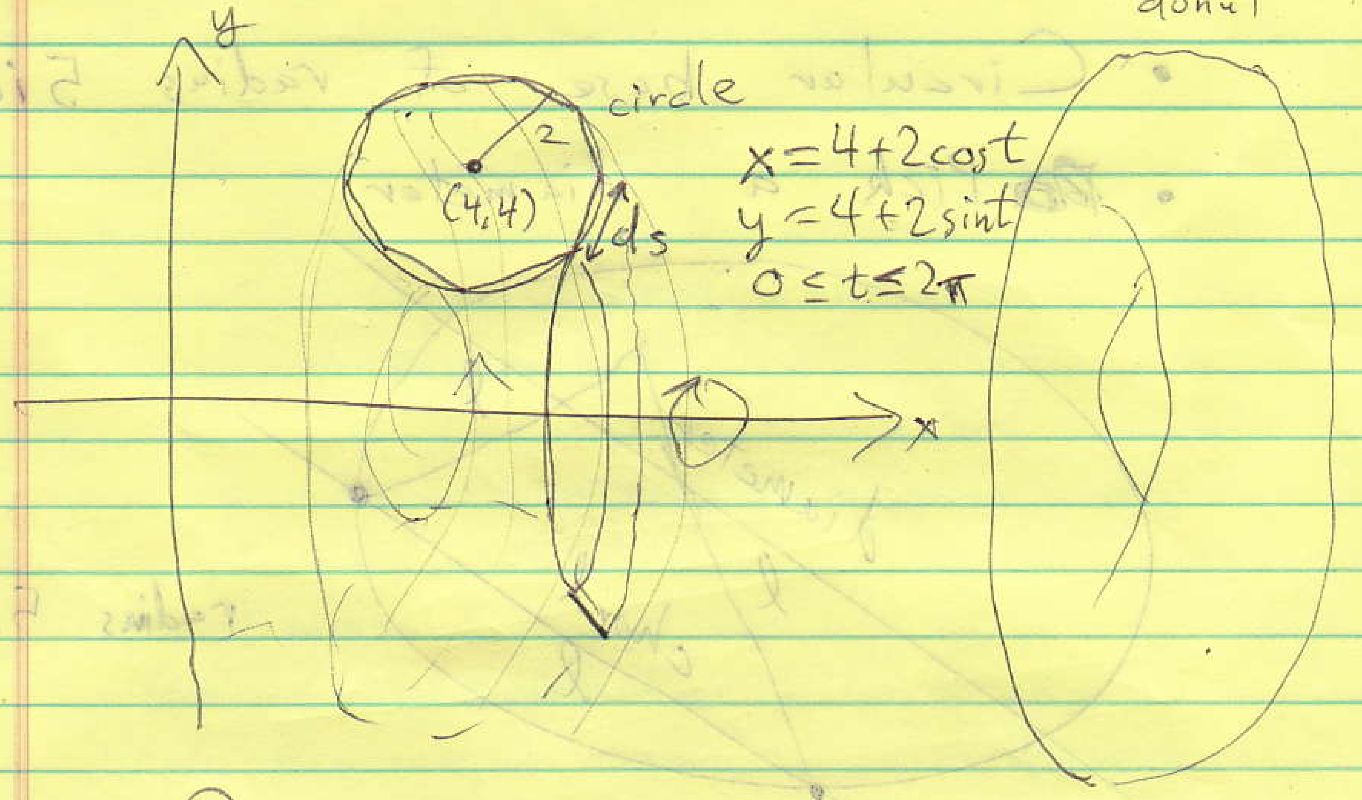
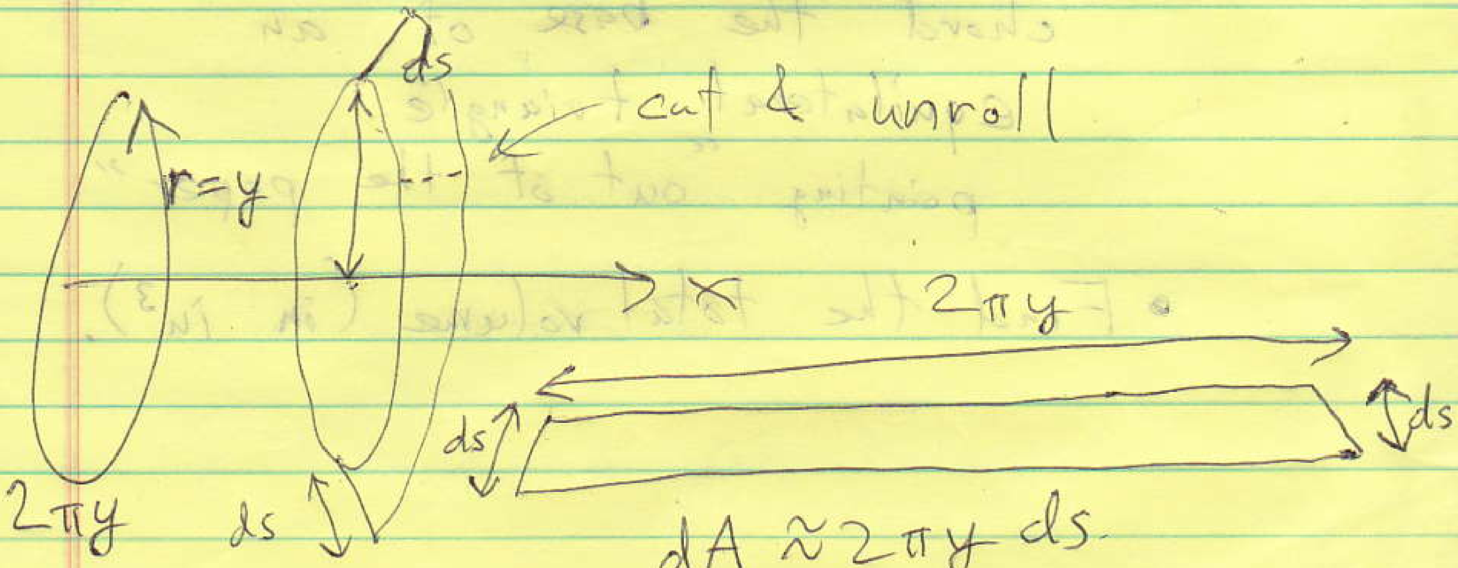
Surface Area = ?



Break curve into short lines:
Estimate area from revolving each line around x-axis:



$dA \approx 2\pi y\, ds$

Total area $= \int_{circle} 2\pi y\, ds$

$x = 4 + 2\cos t$

$y = 4 + 2\sin t$

$0 \le t \le 2\pi$

$ds^2 = dx^2 + dy^2$

$dx = 0 - 2\sin t \, dt$

$dy = 0 + 2\cos t \, dt$

$$A = \int_{t=0}^{t=2\pi} 2\pi y \, ds$$

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{(-2\sin t)^2 dt^2 + (2\cos t)^2 dt^2}$$

$$ds = \sqrt{4\sin^2 t + 4\cos^2 t} \, dt$$

$$ds = \sqrt{4} \, dt = 2 \, dt$$

$$A = \int_0^{2\pi} 2\pi (4 + 2\sin t) \, 2 \, dt$$

$$A = 4\pi \int_0^{2\pi} (4 + 2\sin t) \, dt$$

$$A = 4\pi \left(4t + 2(-\cos t)\right) \Big|_0^{2\pi}$$

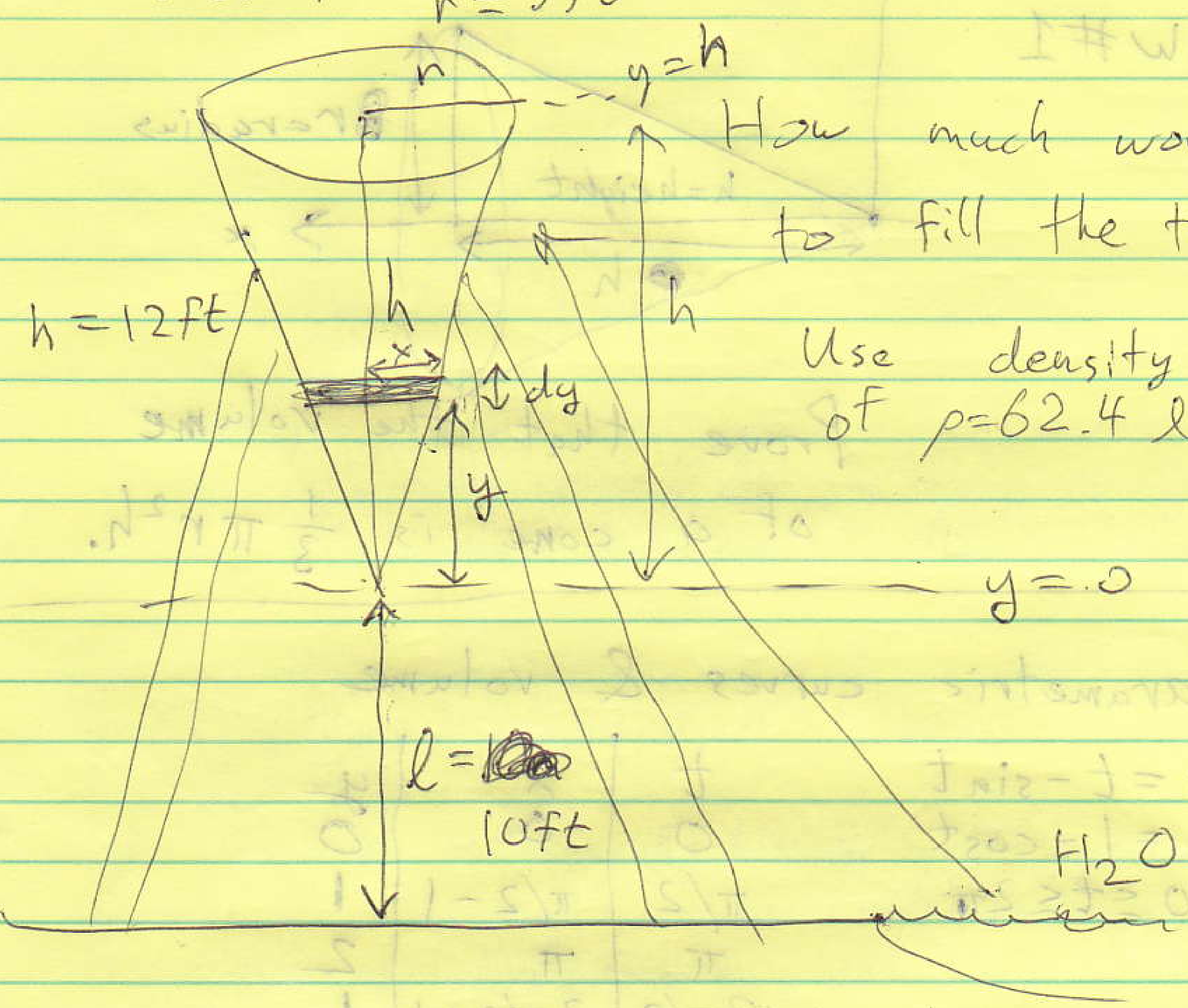
$$A = 4\pi \left(4(2\pi) + 2(-\cos(2\pi))\right) - 4\pi\left(4(0) + 2(-\cos(0))\right)$$

$\cos(2\pi) = 1$, $\cos(0) = 1$

$$A = 4\pi \cdot 4 \cdot 2\pi = 32\pi^2$$

Surface area of donut

Work



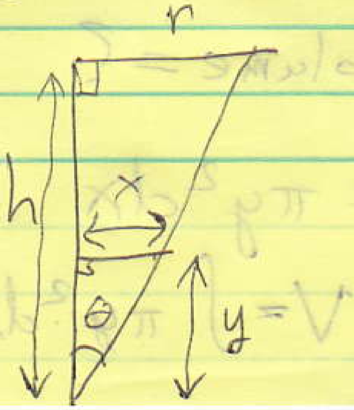
How much work to fill the tank?

Use density of $\rho = 62.4$ lbs/ft^3.

dW = Work for slice = $\underbrace{(\text{weight})}_{\rho \cdot \text{volume}} \times \underbrace{\text{distance}}_{l+y}$

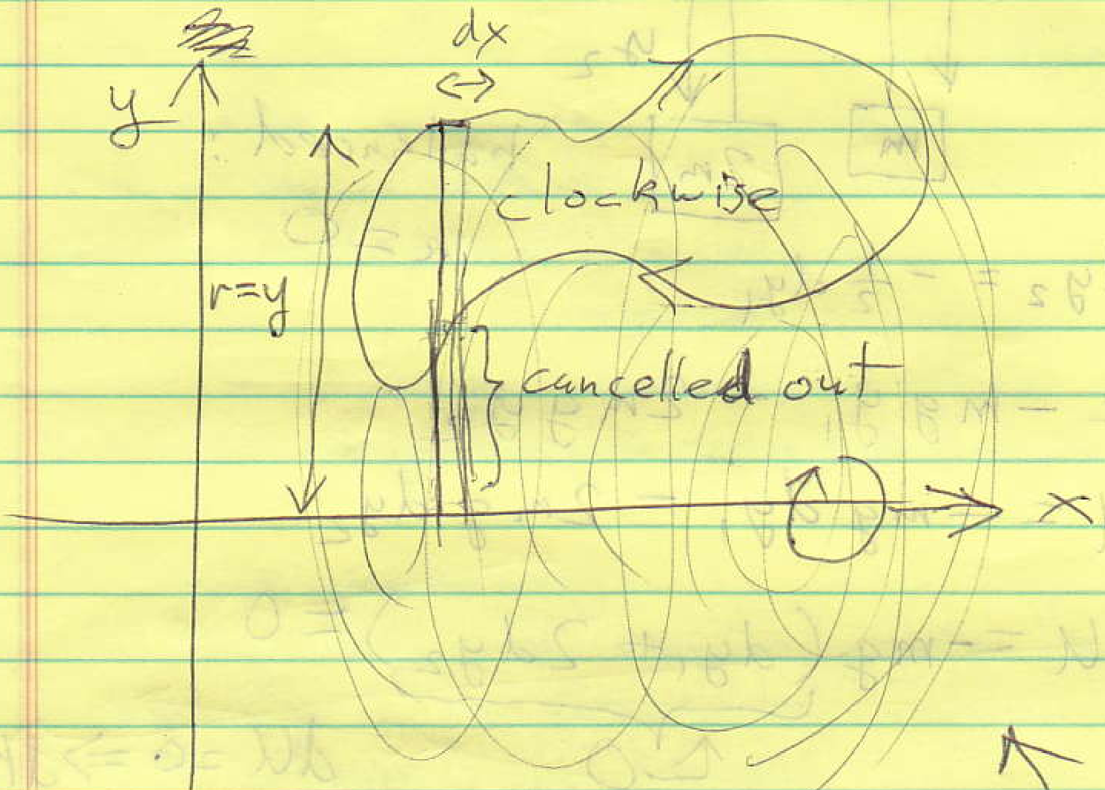
slice: $dW = \rho \cdot \pi x^2 dy \cdot (l+y)$

$x = ?$ $\quad x = \frac{ry}{h}$

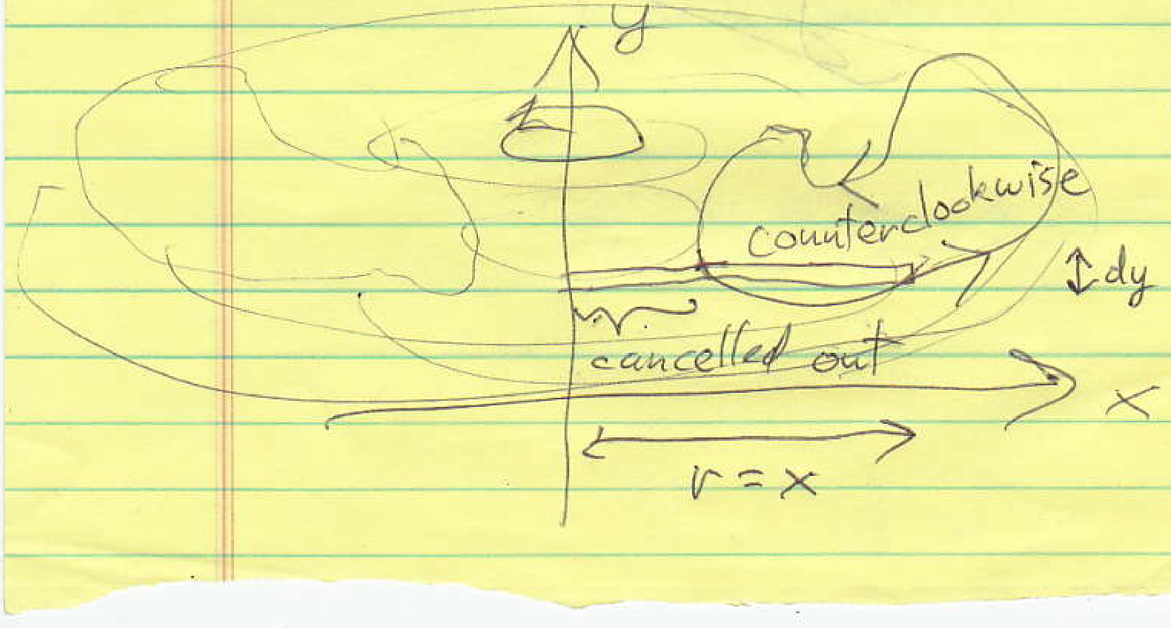


$\tan\theta = \frac{r}{h} = \frac{x}{y}$

$$\boxed{W = \int_{y=0}^{y=h} \rho \pi \left(\frac{ry}{h}\right)^2 dy \, (l+y)}$$

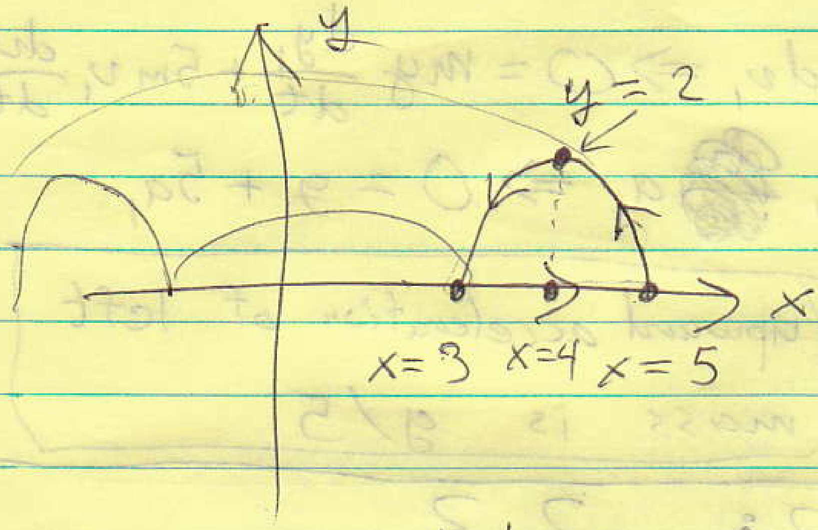


$$V = \int_{\substack{\text{loop} \\ \text{clockwise}}} \pi y^2 \, dx$$



$$V = \int_{\substack{\text{loop} \\ \text{counterclockwise}}} \pi x^2 \, dy$$

Surface area: ~~[illegible]~~ HW

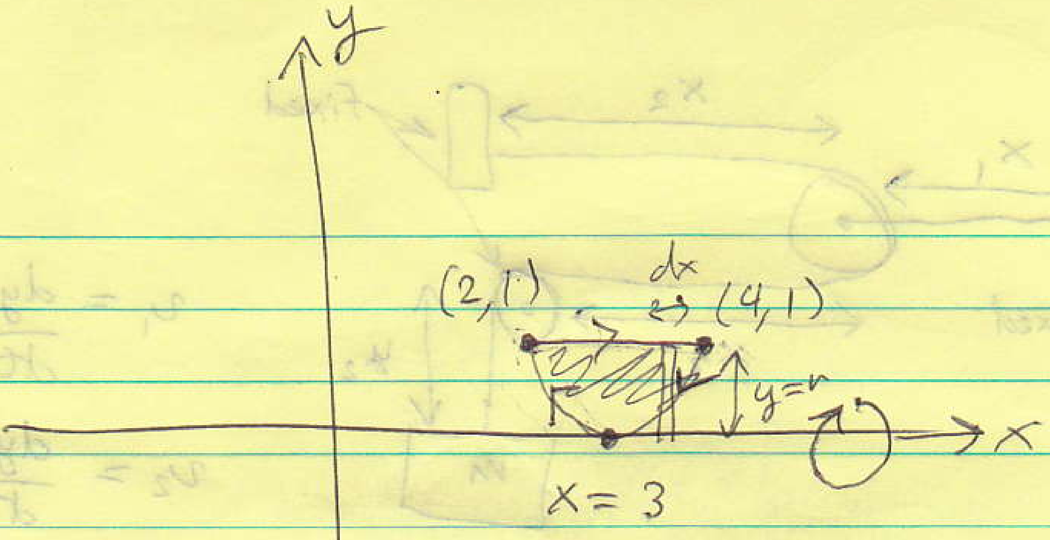


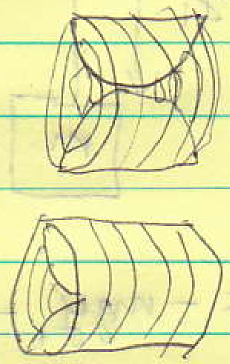
Find a sum of 2 integrals (using just 1 variable) expressing the surface area.

parabola:

$$y = 2 - 2(x-4)^2$$

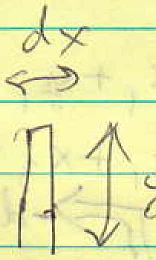
Due Thursday.





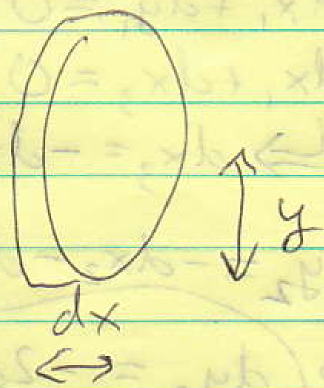
parabola:

$$y = (x-3)^2$$



$$V = \int_{\text{clockwise loop}} \pi y^2 \, dx$$

$$dV = \pi y^2 \, dx$$



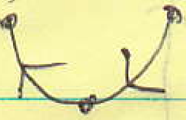
line: $y = 1$

$2 \le x \le 4$

start end

$$\int_{x=2}^{x=4} \pi y^2 \, dx = \int_2^4 \pi 1^2 \, dx = \pi \int_2^4 dx$$

$$= \pi x \Big|_2^4 = \pi(4-2) = 2\pi$$

Parabola: 

$y=(x-3)^2$

$$\int_{x=4}^{x=2} \pi y^2\,dx = \int_4^2 \pi (x-3)^2\,dx$$

$u = x-3 \qquad du = dx$

$x = 2 \Rightarrow u = 2-3 = -1$

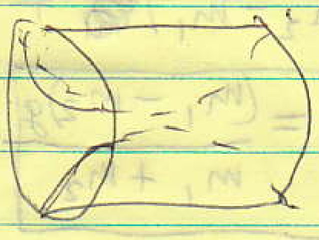
$x = 4 \Rightarrow u = 4-3 = +1$

$$= \int_1^{-1} \pi u^2\,du = \frac{\pi}{3}u^3\Big|_1^{-1} = \frac{\pi}{3}\left((-1)^3 - 1^3\right)$$

$$= -\frac{2\pi}{3}$$

$$V = 2\pi + -\frac{2\pi}{3} = \frac{4\pi}{3}$$

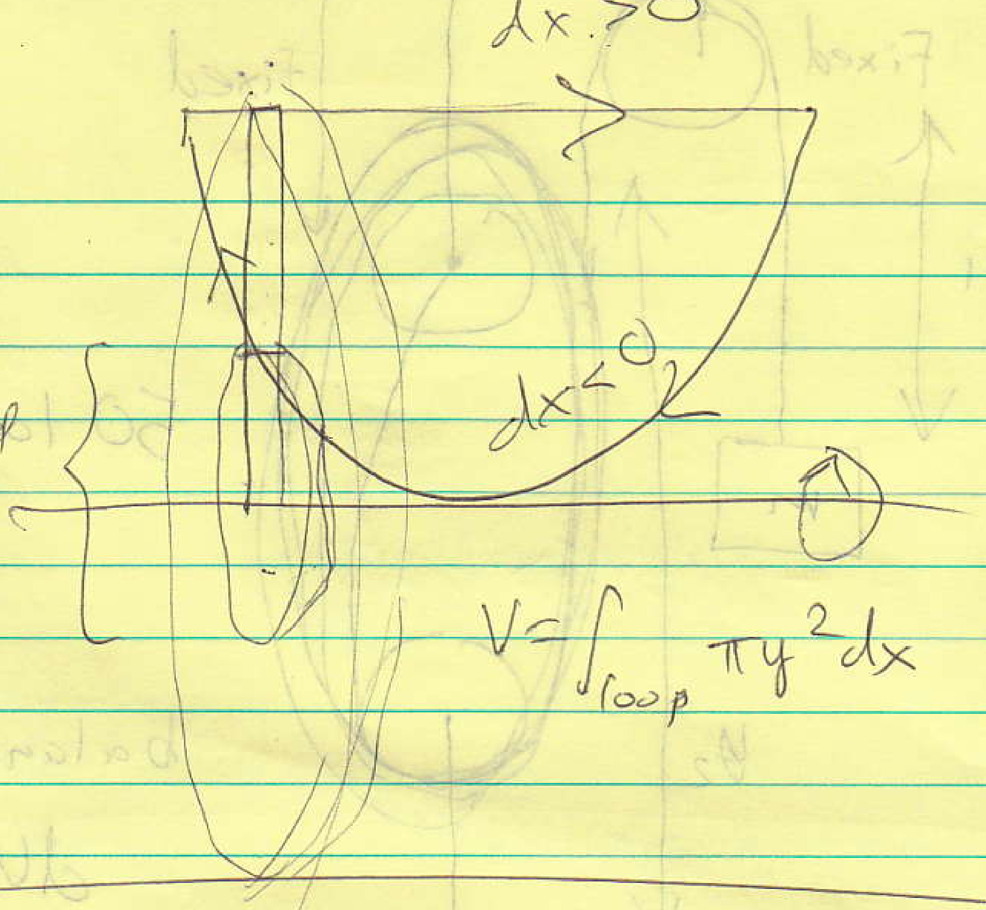
$$\underbrace{\int_{line} \pi y^2\,dx + \int_{parabola} \pi y^2\,dx}_{\int_{loop} \pi y^2\,dx}$$



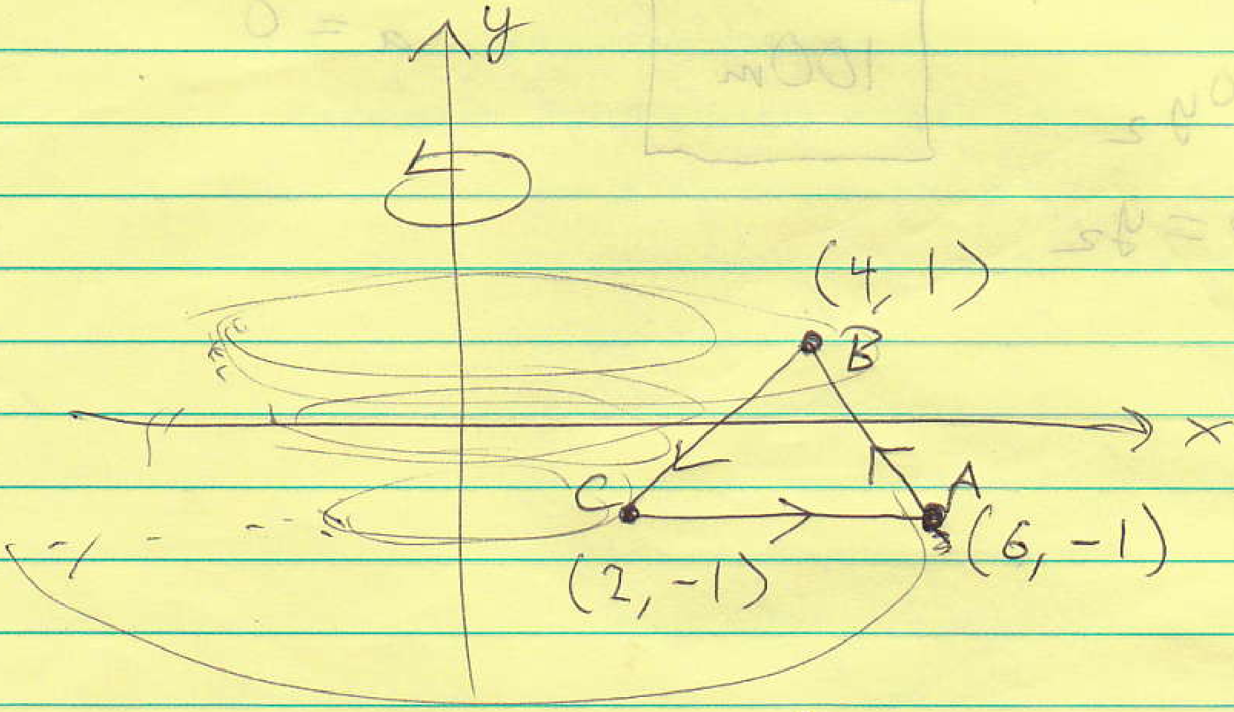
Just setting up the integrals:

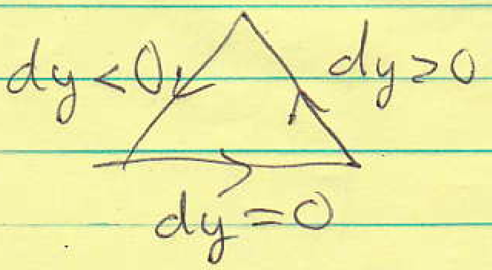
stop at: $V=\int_2^4 \pi \, dx + \int_4^2 \pi(x-3)^2 dx$

cancelled out



$$V = \int_{loop} \pi y^2 \, dx$$





$$V = \int_{loop} \pi x^2 \, dy = \int_A^B \pi x^2 \, dy + \int_B^C \pi x^2 \, dy$$

A to B: $x = A_x(1-t) + B_x t = 6(1-t)+4t$

$y = A_y(1-t) + B_y t = -1(1-t)+1t$

$0 \le t \le 1$

$x = 6-2t$

$y = -1+2t$

$dy = 2dt$

$$\int_A^B \pi x^2\, dy = \boxed{\int_0^1 \pi(6-2t)^2\, 2d}$$