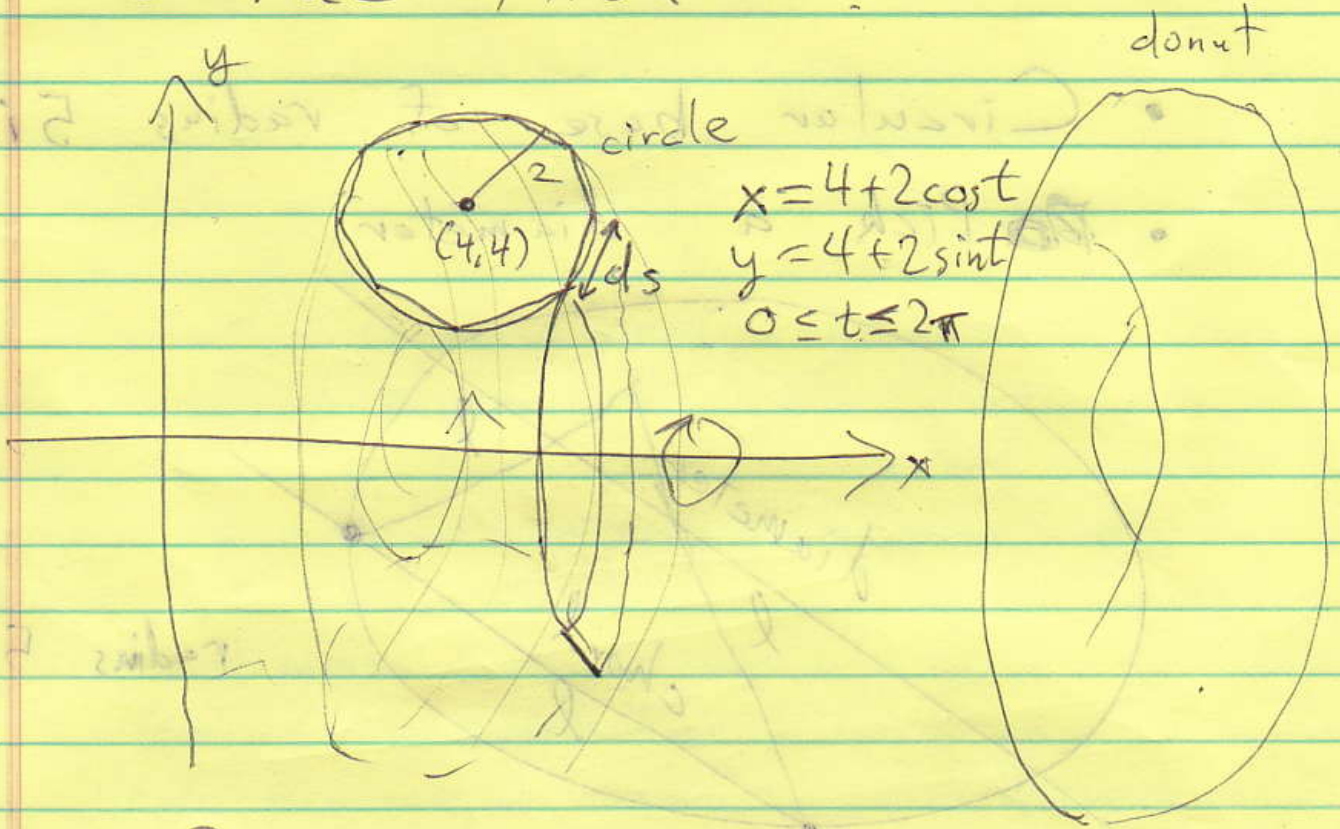
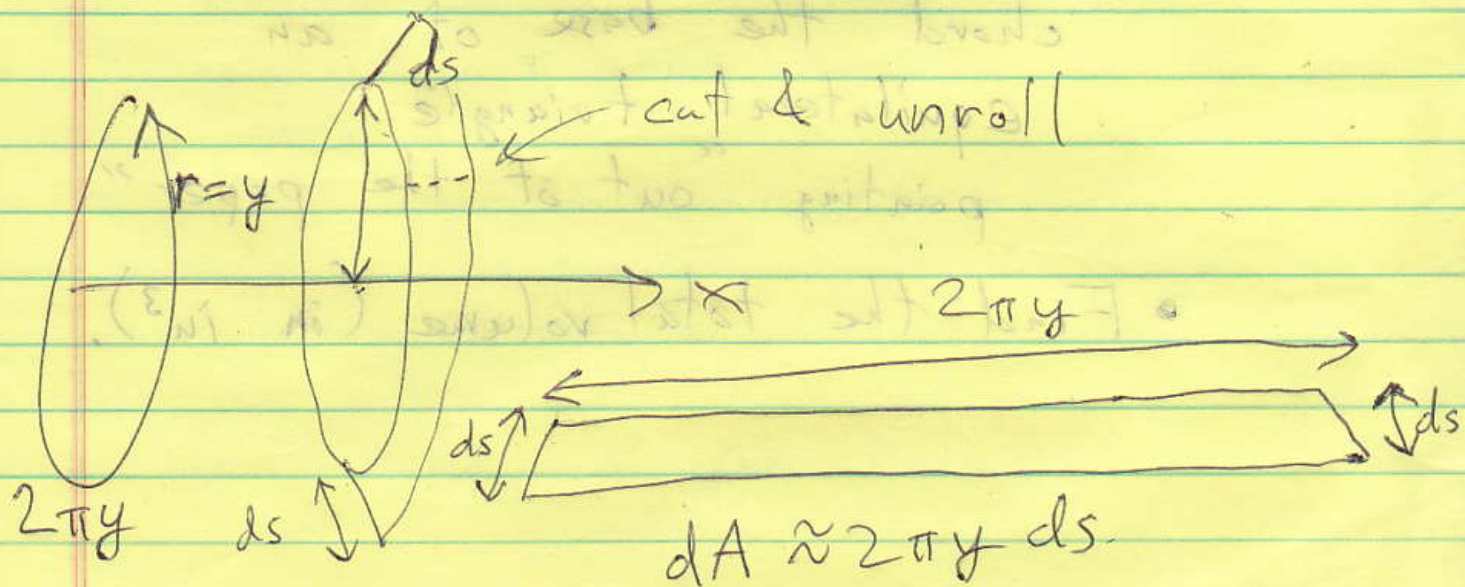


Surface Area = ?



Break curve into short lines:

Estimate area from revolving each line around x-axis:



Total area = $\int_{\text{circle}} 2\pi y ds$

$$\begin{aligned}
 x &= 4 + 2\cos t \\
 y &= 4 + 2\sin t \\
 0 &\leq t \leq 2\pi
 \end{aligned}
 \quad
 \begin{aligned}
 ds^2 &= dx^2 + dy^2 \\
 dx &= 0 - 2\sin t \, dt \\
 dy &= 0 + 2\cos t \, dt
 \end{aligned}$$

$$A = \int_{t=0}^{t=2\pi} 2\pi y \, ds$$

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{(-2\sin t)^2 dt^2 + (2\cos t)^2 dt^2}$$

$$ds = \sqrt{4\sin^2 t + 4\cos^2 t} \, dt$$

$$ds = \sqrt{4} \, dt = 2 \, dt$$

$$A = \int_0^{2\pi} 2\pi (4 + 2\sin t) 2 \, dt$$

$$A = 4\pi \int_0^{2\pi} (4 + 2\sin t) \, dt$$

$$A = 4\pi (4t + 2(-\cos t)) \Big|_0^{2\pi}$$

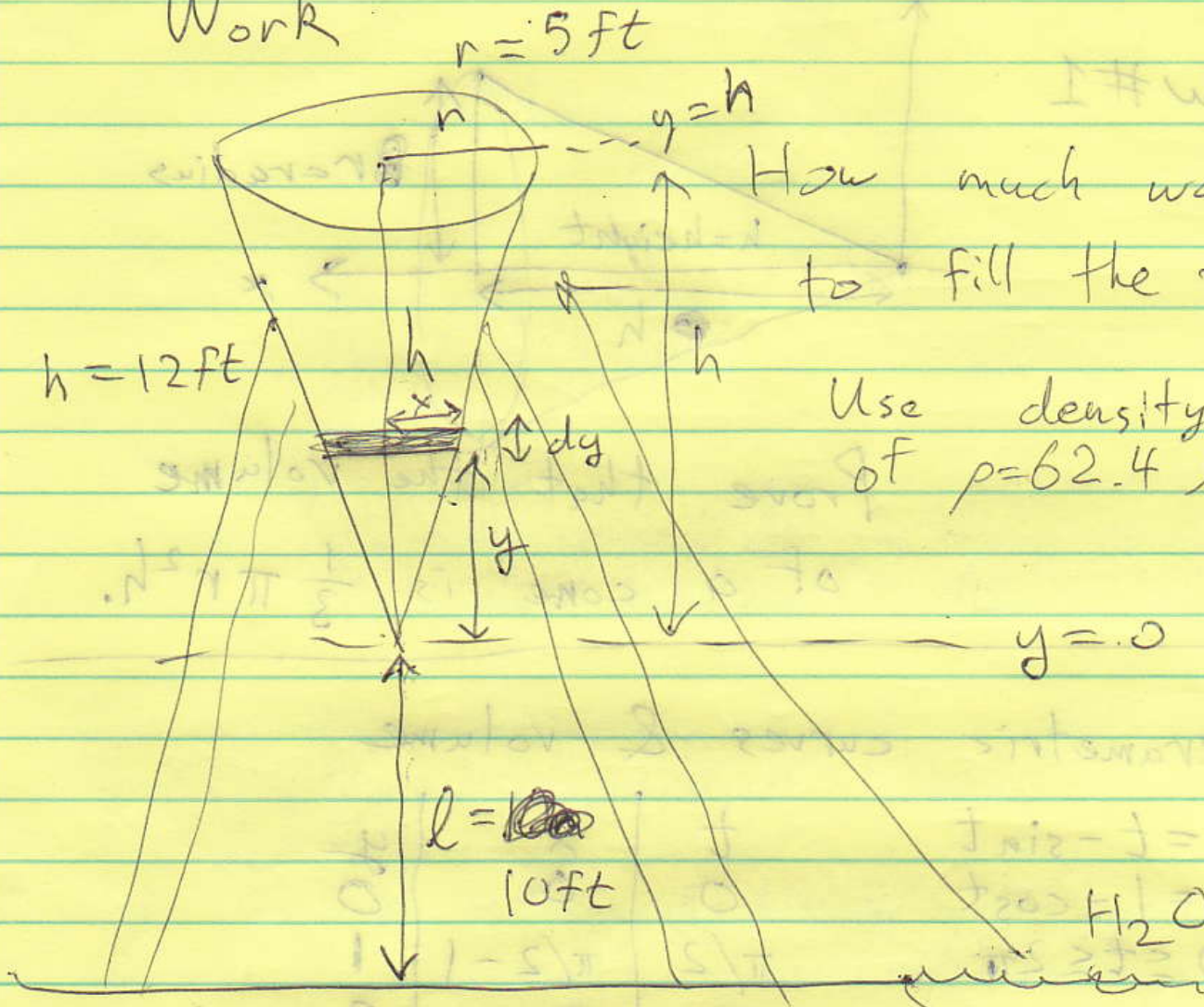
$$A = 4\pi (4(2\pi) + 2(-\cos(2\pi)))$$

$$- 4\pi (4(0) + 2(-\cos(0)))$$

$$A = 4\pi \cdot 4 \cdot 2\pi = 32\pi^2$$

Surface area of donut

Work



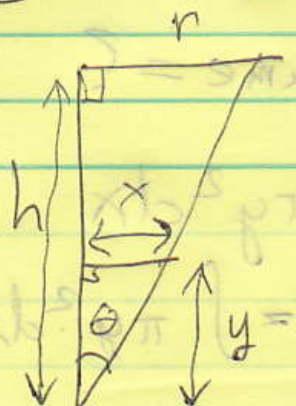
How much work to fill the tank?

Use density of $\rho = 62.4 \text{ lbs/ft}^3$.

$dW = \text{Work for slice} = (\text{weight}) \times \text{distance}$

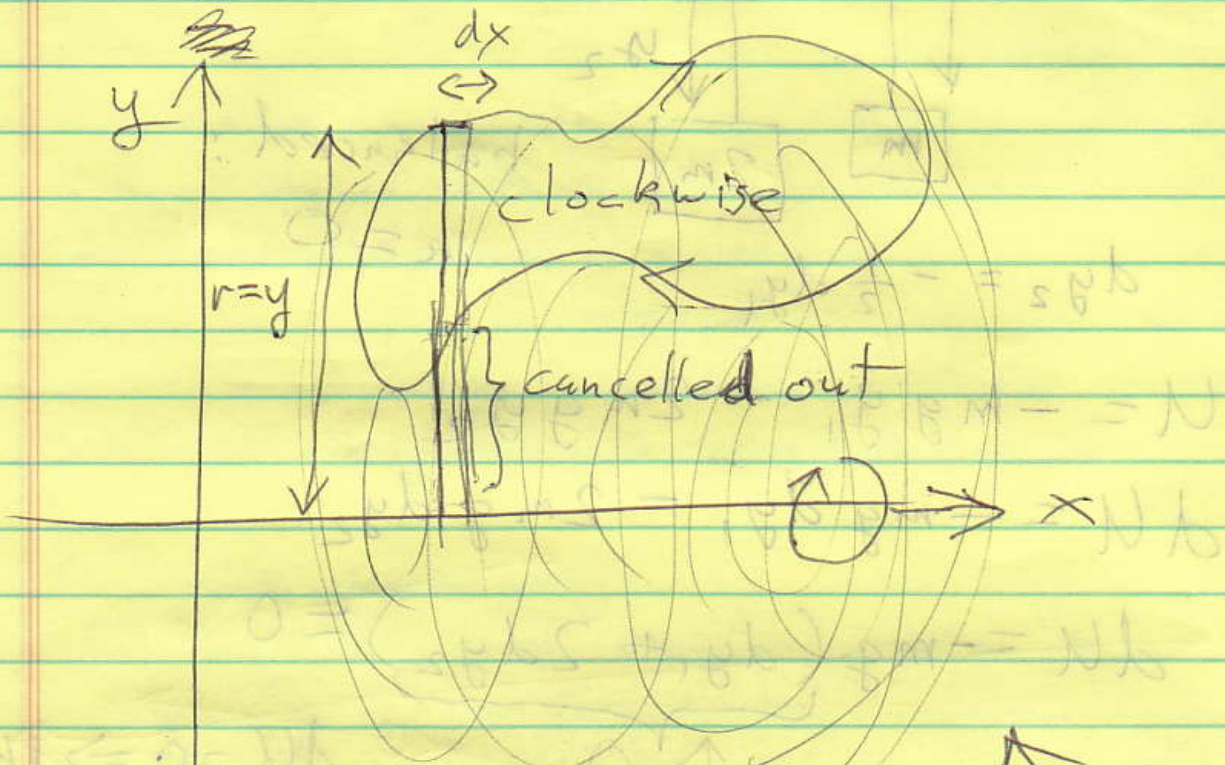


$dW = \rho \cdot \text{volume} \cdot (l + y)$
 $dW = \rho \cdot \pi x^2 dy \cdot (l + y)$



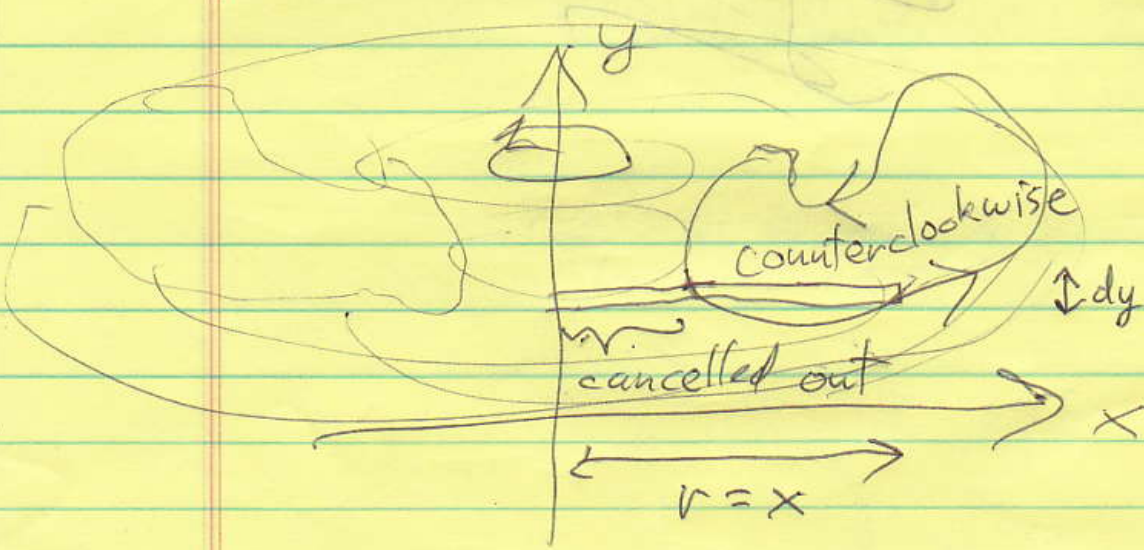
$x = ? \quad x = \frac{ry}{h}$
 $\tan \theta = \frac{r}{h} = \frac{x}{y}$

$$W = \int_{y=0}^{y=h} \rho \pi \left(\frac{ry}{h} \right)^2 dy (l+y)$$



$$V = \int_{\text{loop}} \pi y^2 dx$$

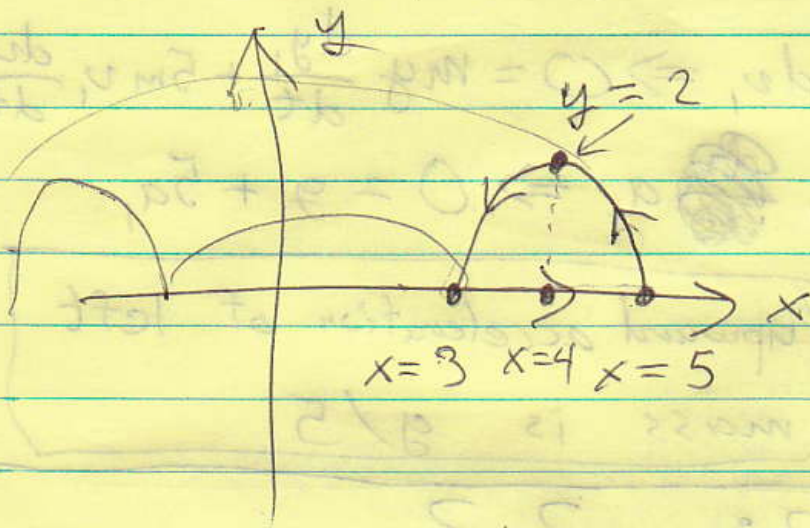
clockwise



$$V = \int_{\text{loop}} \pi x^2 dy$$

counterclockwise

Surface area ~~Parabola~~ HW

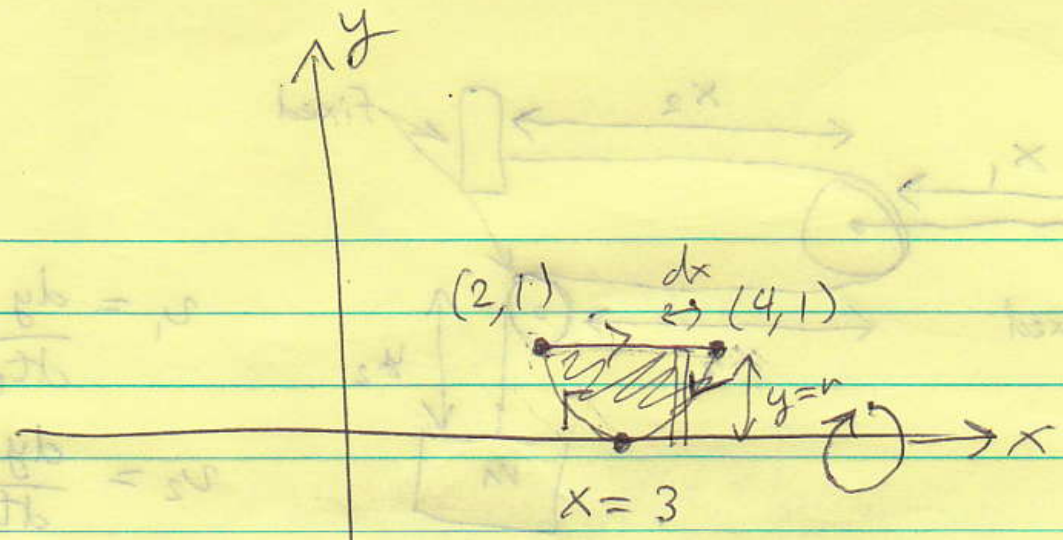


Find a sum of
2 integrals
(using just 1
variable)
expressing the
surface area

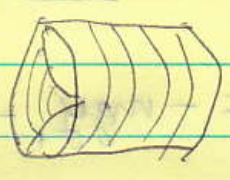
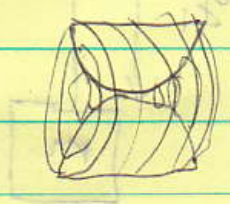
parabola:

$$y = 2 - 2(x - 4)^2$$

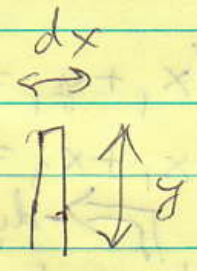
Due Thursday.



parabola:
 $y = (x-3)^2$

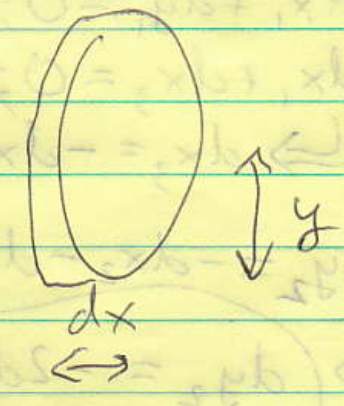


$$V = \int_{\text{clockwise loop}} \pi y^2 dx$$



line: $y = 1$
 $2 \leq x \leq 4$
 ↑ start ↑ end

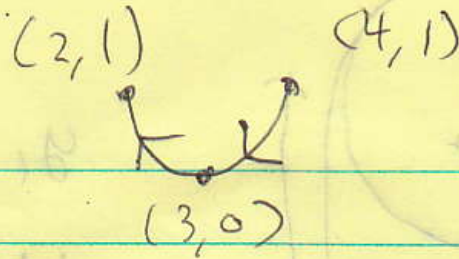
$$dV = \pi y^2 dx$$



$$\int_{x=2}^{x=4} \pi y^2 dx = \int_2^4 \pi 1^2 dx = \pi \int_2^4 dx$$

$$= \pi x \Big|_2^4 = \pi(4-2) = 2\pi$$

Parabola:



$$y = (x-3)^2$$

$$\int_{x=4}^{x=2} \pi y^2 dx = \int_4^2 \pi (x-3)^2 dx$$



$$u = x-3 \quad du = dx$$

$$x=2 \Rightarrow u = 2-3 = -1$$

$$x=4 \Rightarrow u = 4-3 = +1$$

$$= \int_1^{-1} \pi u^2 du = \frac{\pi}{3} u^3 \Big|_1^{-1} = \frac{\pi}{3} ((-1)^3 - 1^3) = -\frac{2\pi}{3}$$

$$V = 2\pi + -\frac{2\pi}{3} = \frac{4\pi}{3}$$

$$\int_{\text{line}} \pi y^2 dx + \int_{\text{parabola}} \pi y^2 dx$$

$$\int_{\text{loop}} \pi y^2 dx$$



Just setting up the integrals:

stop at: $V = \int_2^4 \pi dx + \int_4^2 \pi(x-3)^2 dx$

b) The kinetic energy decreased (deceleration)
so magnetic energy increased

#10 $\frac{\Delta E}{\Delta t} = 500W$ $\Delta E = mc\Delta T$

$m = 60kg$ $c = 4.186 \frac{J}{kg \cdot ^\circ C}$ $\Delta T = 8^\circ C$

$\Delta E = (60kg) \left(\frac{10^3 J}{kg} \right) (8^\circ C) = 4.8 \times 10^5 J$

$\Delta t = \frac{\Delta E}{500W} = \frac{4.8 \times 10^5 J}{500 \frac{J}{s}} = 9.6 \times 10^2 s$

$\Delta t = 9.6 \times 10^2 s = \frac{9.6 \times 10^2 s}{60 \frac{s}{min}} = 16 \text{ minutes}$

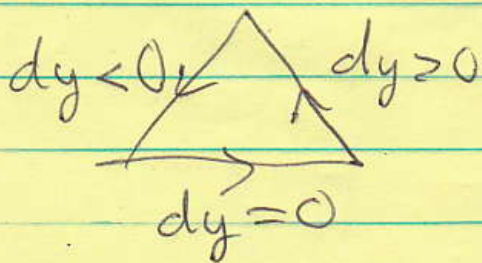
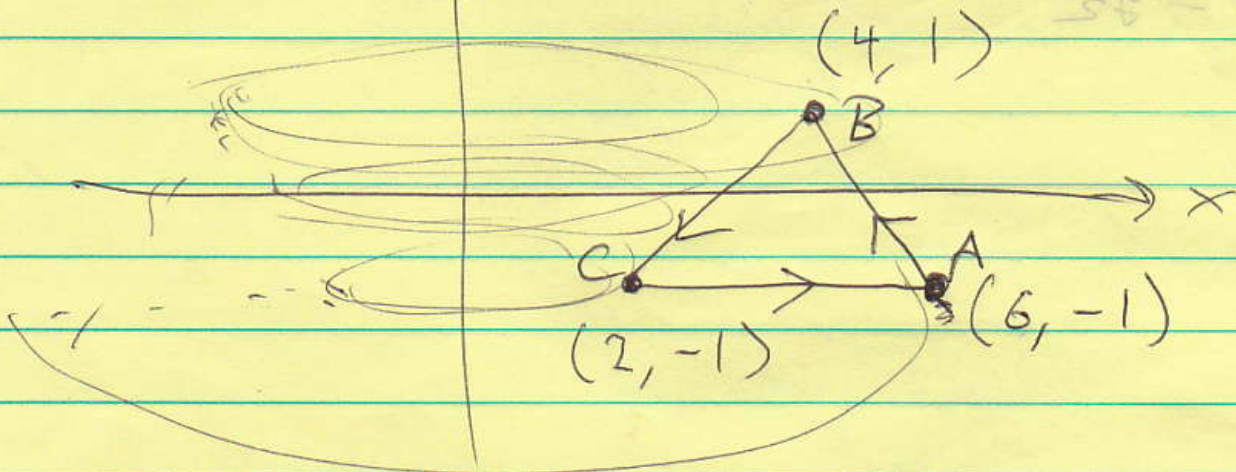
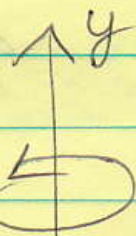
(about 2 hours)

cancelled
out

$$dx > 0$$

$$dx < 0$$

$$V = \int_{\text{loop}} \pi y^2 dx$$

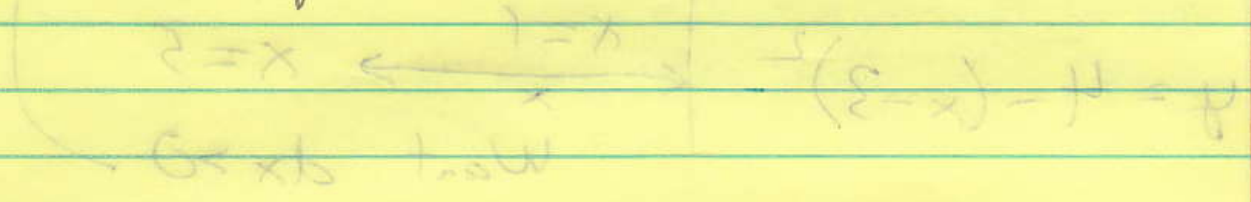


$$V = \int_{\text{loop}} \pi x^2 dy = \int_A^B \pi x^2 dy + \int_B^C \pi x^2 dy$$

A to B: $x = A_x(1-t) + B_x t = 6(1-t) + 4t$
 $y = A_y(1-t) + B_y t = -1(1-t) + 1t$
 $0 \leq t \leq 1$

$x = 6 - 2t$
 $y = -1 + 2t$
 $dy = 2dt$

$\int_A^B \pi x^2 dy = \int_0^1 \pi (6-2t)^2 2dt$



$V = \int_{x=1}^{x=2} \pi x^2 dx$

$V = \int_1^2 \pi x^2 dx$

should be πx^2

still the way work too

