Today: substitution

$$\int_0^1 \pi(2-t)^2\,dt$$

w/o substitution:

$$\int_0^1 \pi(4-4t+t^2)\,dt$$

$$\pi\left(4t-2t^2+\frac{t^3}{3}\right)\Big|_0^1$$

$$=\pi(4-2+1/3)-\pi(0-0+0)$$

$$=\pi(7/3)\ \checkmark$$

with substitution: $u=2-t$

$$\int_{0=t}^{1=t} \pi u^2\,dt \qquad du=-dt$$

different variables

$t=1 \Rightarrow u=2-1=1$

$t=0 \Rightarrow u=2-0=2$

$$\int_2^1 \pi u^2(-du)=\int_1^2 \pi u^2\,du$$

$$=\frac{\pi}{3}u^3\Big|_1^2=\frac{\pi}{3}(8-1)=\frac{7\pi}{3}\ \checkmark$$

$$\int_0^1 \pi (2-t)^{57}\, dt$$

$$(t-2)^{57} = -2^{57} + 57 \cdot 2^{56} t - \frac{57 \cdot 56}{2} \cdot 2^{55} t^2 + \underbrace{\cdots}_{54 \text{ terms}} + t^{57}$$

$u = 2 - t \qquad du = (2-t)' dt = (-1) dt$

$t = 1 \Rightarrow u = 2 - 1 = 1 \qquad dt = -du$

$t = 0 \Rightarrow u = 2 - 0 = 2$

$$\int_0^1 \pi (2-t)^{57} dt = \int_2^1 \pi u^{57} (-du)$$

$$= \int_1^2 \pi u^{57} du = \frac{\pi}{58} u^{58} \Big|_1^2$$

$$= \frac{\pi}{58}\left(2^{58} - 1^{58}\right) = \frac{\pi}{58}\left(2^{58} - 1\right)$$

$$\int \frac{dx}{x \ln^3 x} = \int \underbrace{(\ln x)}_{u}^{-3} \frac{dx}{x}$$

$$du = (\ln x)' dx = \frac{1}{x} dx = \frac{dx}{x}$$

$$= \int u^{-3} du = \frac{u^{-3+1}}{-3+1} + c = \frac{u^{-2}}{-2} + c$$

$$= -\frac{1}{2u^2} + c = \boxed{\frac{-1}{2\ln^2 x} + c}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad (\text{if } n \neq -1)$$

$$\int x^{-1}\,dx = \ln|x| + c$$

$$\int \frac{10+4x}{3+5x+x^2}\,dx$$

$u = 3+5x+x^2$

$du = (0+5+2x)\,dx$

$2\,du = (10+4x)\,dx$

$$= \int \frac{2\,du}{u}$$

$$= 2\ln|u| + c = 2\ln|3+5x+x^2| + c$$

$$\int \underbrace{\cos^7(x)}_{(\cos x)^7} \overbrace{\sin(x)\,dx}^{-du}$$

$$(\underbrace{\cos x}_{u})^7$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$\int u^7(-du) = -\frac{u^8}{8} + c$$

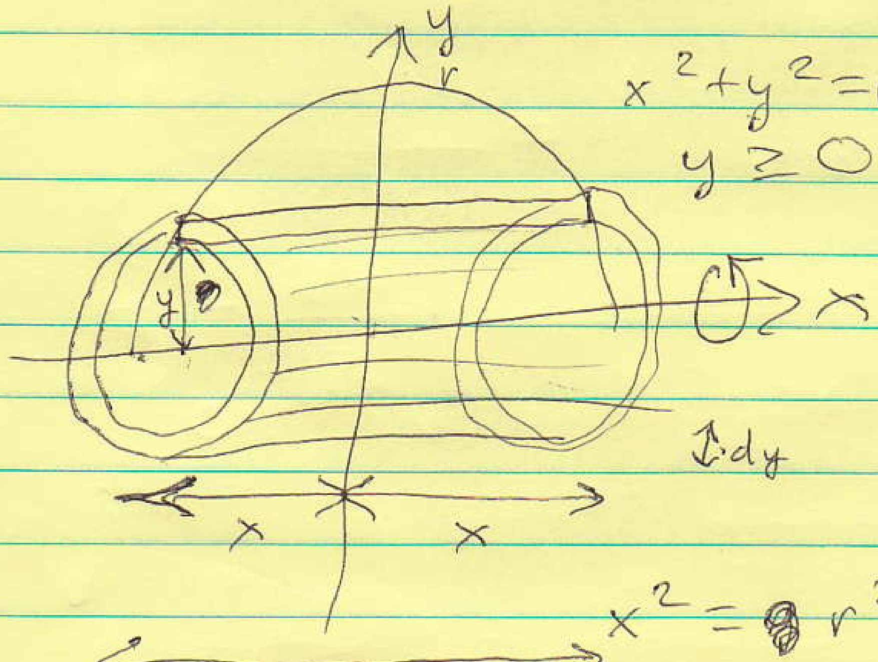
$$= \boxed{-\frac{\cos^8 x}{8} + c}$$

HW #1 $\displaystyle\int_{\pi/4}^{\pi/3} \sin^5 x \cos x \, dx = ?$

#2 $\displaystyle\int_2^3 \frac{dx}{x \ln x} = ?$

#3 $\displaystyle\int (5x^2+1)^{10} x \, dx = ?$

Prove that the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$



$x^2+y^2=r^2$

$y \geq 0$

$2\sqrt{r^2-y^2}$

$x^2 = r^2-y^2$

$x = \sqrt{r^2-y^2}$

shell: $dV = 2\sqrt{r^2-y^2}\,(2\pi y)\,dy$

$$V = \int_0^r 2\underbrace{\sqrt{r^2-y^2}}\overbrace{(2\pi y)\,dy}^{-\pi\,du}$$

$y = r \Rightarrow u = r^2 - r^2 = 0$ | $u = r^2 - y^2$

$y = 0 \Rightarrow u = r^2 - 0^2 = r^2$ | $du = (0 - 2y)\,dy$

$-\pi\,du = 2\pi y\,dy$

$$V = \int_{r^2}^{0} 2\sqrt{u}\,(-\pi\,du) = \int_0^{r^2} 2\sqrt{u}\,\pi\,du$$

$$V = 2\pi \int_0^{r^2} u^{1/2}\,du = 2\pi \left.\frac{u^{1/2+1}}{1/2+1}\right|_0^{r^2}$$

$$= 2\pi \left.\frac{u^{3/2}}{3/2}\right|_0^{r^2} = 2\pi\left(\frac{2}{3}\right)u^{3/2}\Big|_0^{r^2}$$

$$= \frac{4\pi}{3}\left((r^2)^{3/2} - 0^{3/2}\right) = \frac{4\pi}{3}r^3 \checkmark$$

HW #4) $\int_0^1 x\sqrt{1-x^2}\,dx = ?$

#5) $\int_0^2 x^3\sqrt{x^2+4}\,dx = ?$

$$I = \int_{\pi/12}^{\pi/6} \left[\underbrace{\sqrt[3]{\tan(2x)}}_{u} \Big/ \cos^2(2x) \right] dx$$

$$du = \sec^2(2x) \cdot 2\,dx$$

$$du = d(\tan 2x) = \sec^2(2x)\, d(2x)$$
$$= \sec^2(2x) \cdot 2\,dx$$
$$= \frac{2\,dx}{\cos^2 2x}$$

x-limit

$$I = \int_{\pi/12}^{\pi/6} \sqrt[3]{u}\ \frac{dx}{\cos^2 2x}$$

x-limit

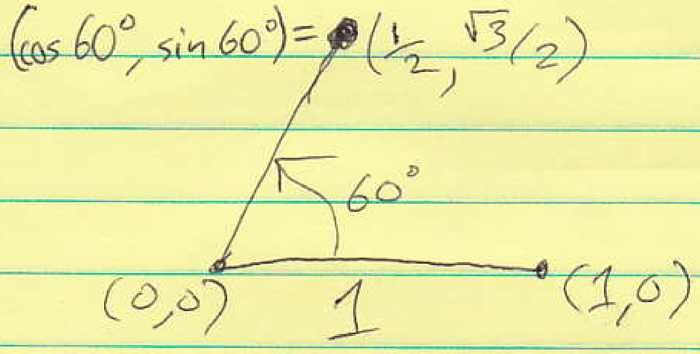
$du/2$

$$x = \pi/6 \Rightarrow u = \tan(\pi/3) = \frac{\sin \pi/3}{\cos \pi/3}$$

$$= \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\sqrt{3}/2}{1/2}$$

$$\Downarrow$$

$$u = \sqrt{3}$$



$$x = \pi/12 \Rightarrow u = \tan(\pi/6) = \frac{\sin \pi/6}{\cos \pi/6}$$

$$= \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$I = \int_{1/\sqrt{3}}^{\sqrt{3}} \sqrt[3]{u} \cdot du/2 = \frac{1}{2}\int_{1/\sqrt{3}}^{\sqrt{3}} u^{1/3}\, du$$

$$= \frac{1}{2}\left(\frac{u^{1/3+1}}{1/3+1}\right)\Bigg|_{1/\sqrt{3}}^{\sqrt{3}} = \frac{1}{2}\,\frac{u^{4/3}}{4/3}\Bigg|_{1/\sqrt{3}}^{\sqrt{3}}$$

$$= \frac{1}{2}\cdot\frac{3}{4}\left(\sqrt{3}^{\,4/3} - \left(\frac{1}{\sqrt{3}}\right)^{4/3}\right)$$

$$= \frac{3}{8}\left(\left(3^{1/2}\right)^{4/3} - \left(3^{-1/2}\right)^{4/3}\right)$$

$$= \frac{3}{8}\left(3^{2/3} - 3^{-2/3}\right)$$

$$= \frac{3}{8}\left(\sqrt[3]{9} - 1/\sqrt[3]{9}\right)$$