$$V = \int_0^{2\pi} \pi (4 + 3\sin t)^2 (2 \sin t \, dt).$$

Find the exact value of V.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$V = 2\pi \int_0^{2\pi} (4^2 + 2 \cdot 4 \cdot 3 \sin t + (3 \sin t)^2)$$
$$\cdot \sin t \, dt$$

$$V = 2\pi \int_0^{2\pi} (16 \sin t + 24 \sin^2 t + 9 \sin^3 t) dt$$

$$V = 2\pi (A + B + C)$$

$$A = \int_0^{2\pi} 16 \sin t \, dt = -16 \cos t \Big|_0^{2\pi}$$

$$B = \int_0^{2\pi} 24 \sin^2 t \, dt$$

$$C = \int_0^{2\pi} 9 \sin^3 t \, dt$$

Note: $\sin^3 t = (\sin t)^3$

$$A = -16 \cos 2\pi - (-16 \cos 0)$$
$$= -16(1) - (-16(1)) = 0$$

$$B = \int_0^{2\pi} 24 \sin^2 t \, dt$$

Use a half angle formula:

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

(There's also $\cos^2 x = \frac{1 + \cos 2x}{2}$.)

$$B = \int_0^{2\pi} 24 \left(\frac{1 - \cos 2t}{2} \right) dt$$

$$= 12 \int_0^{2\pi} (1 - \cos \underbrace{2t}) \, dt$$

$t = 0 \Rightarrow u = 0$

$t = 2\pi \Rightarrow u = 4\pi$

$u = 2t$

$du = 2\,dt$

$du/2 = dt$

$$B = 12 \int_0^{4\pi} (1 - \cos u)(du/2)$$

$$= 6 \int_0^{4\pi} (1 - \cos u) \, du$$

$$= 6(u - \sin u)\Big|_0^{4\pi} = 6(4\pi - \underbrace{\sin 4\pi}_{=0}) - \underbrace{6(0 - \sin 0)}_{=0}$$

$$\boxed{B = 24\pi}$$

In general, if you have $\int \cos^{2n} x \, dx$, change it

to $$\int \underbrace{\left(\underbrace{\frac{1+\cos 2x}{2}}_{\cos^2 x}\right)^n}_{(\cos^2 x)^n = \cos^{2n} x} dx$$

$$\int \cos^4 x \, dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \int \frac{1^2 + 2 \cdot 1 \cdot \cos 2x + \cos^2 2x}{2^2} \, dx$$

$$= \int \left(\frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x\right) dx$$

$$= \frac{1}{4}x + \int \left(\frac{1}{2}\cos u + \frac{1}{4}\cos^2 u\right) \frac{du}{2}$$

using $u = 2x \Rightarrow du = 2dx$

$\Rightarrow dx = du/2$

$$= \frac{1}{4}x + \frac{1}{4}\sin u + \frac{1}{8}\int \frac{1+\cos 2u}{2}\,du$$

$$= \frac{1}{4}x + \frac{1}{4}\sin u + \frac{1}{8}\int \frac{1+\cos w}{2}\,\frac{dw}{2}$$

$w = 2u \quad dw = 2du \quad dw/2 = du$

$$= \frac{1}{4}x + \frac{1}{4}\sin u + \frac{1}{3\cdot 2}(w + \sin w) + c$$

$$= \frac{1}{4}x + \frac{1}{4}\sin u + \frac{1}{32}(2u + \sin 2u) + c$$

$$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{32}(4x + \sin 4x) + c$$

This is $\int \cos^4 x\,dx$

HW #1: Find $\int \sin^4 x\,dx$

Find $C = \int_0^{2\pi} 9\sin^3 t\,dt$

$$C = 9\int_0^{2\pi} \underbrace{\sin^2 t}_{=1-\cos^2 t}\,\sin t\,dt$$

because $\cos^2 t + \sin^2 t = 1$

$\sin^2 t = 1 - \cos^2 t$

$$C = 9\int_0^{2\pi} (1-\cos^2 t)\,\underbrace{\sin t\, dt}$$

$u = \cos t \Rightarrow du = -\sin t\, dt$

$-du = \sin t\, dt$

$t = 2\pi \Rightarrow u = \cos 2\pi = 1$

$t = 0 \Rightarrow u = \cos 0 = 1$

$$C = 9\int_1^1 (1-u^2)(-du) = 0$$

same

~~$V = A + B + C = 0 + 24\pi + 0 = 24\pi$~~

$$V = 2\pi(A+B+C) = 2\pi(0 + 24\pi + 0)$$

$$= 48\pi^2$$

HW #2 Find the exact volume

$$V = \int_0^{2\pi} \pi(2+\cos t)^2 \cos t\, dt$$

(from test 3, #2).

$$\int \cos^{2m+1} x \, dx = \int \cos^{2m} x \cos x \, dx$$

$$= \int (\cos^2 x)^m \cos x \, dx$$

$$= \int (1-\sin^2 x)^m \cos x \, dx$$

$u = \sin x \qquad du = \cos x \, dx$

$$= \int (1-u^2)^m \, du$$

E.g. $\displaystyle\int \cos^5 x \, dx = \int (1-u^2)^2 \, du$

$5 = 2 \cdot 2 + 1$

$$= \int (1 - 2u^2 + u^4) \, du = u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + c$$

$$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$$

HW #3 Find $\displaystyle\int_0^{\pi/4} \sin^5 x \cos^2 x \, dx$

$\displaystyle\int \sin^{3} x \cos^4 x \, dx = \int \sin^2 x \sin x \cos^4 x \, dx$ ← odd

$$= \int (1-\cos^2 x) \sin x \cos^4 x \, dx$$

$u = \cos x \qquad du = -\sin x \, dx$

$-du = \sin x \, dx$

$$\int \underbrace{(1-\cos^2 x)}_{1-u^2} \underbrace{\cos^4 x}_{u^4} \underbrace{\sin x \, dx}_{-du}$$

$$= \int (1-u^2) u^4 (-du) = \int (u^2-1) u^4 \, du$$

$$= \int (u^6 - u^4) \, du = \frac{u^7}{7} - \frac{u^5}{5} + c$$

$$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$$

Odd power of sine or cosine:

Use $\sin^{2m+1} x = (1-\cos^2 x)^m \sin x$

$\cos^{2m+1} x = (1-\sin^2 x)^m \cos x$

Only even powers of sine or cosine:

Use $\cos^{2m} x = \left(\frac{1+\cos 2x}{2}\right)^m$

$\sin^{2m} x = \left(\frac{1-\cos 2x}{2}\right)^m$