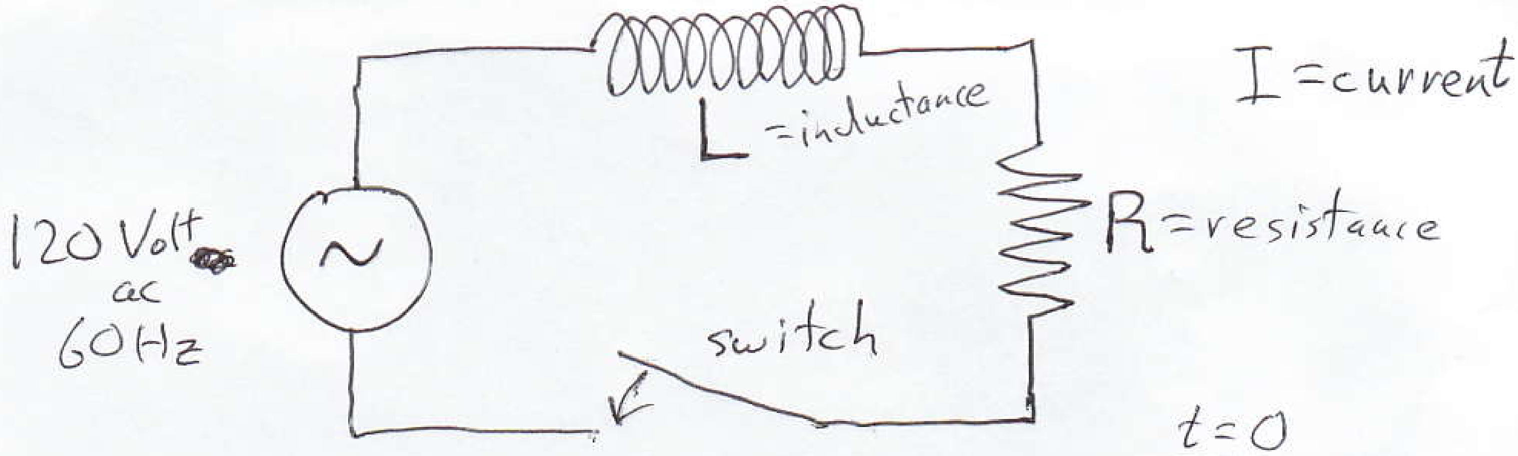
Today more linear differential equations.
(14.2, 14.3)

Tomorrow: review and your questions

Monday: Test 5 (notes; no calculator)



What happens when we $\overbrace{\text{close the switch}}^{t=0}$?

$$L \cdot \frac{dI}{dt} + RI = \underbrace{V(t)}_{\text{voltage}} = (120 \text{ volt}) \cos(\omega t + \phi)$$

$\omega = 2\pi \times 60$ radians/second $= 120\pi$/second

t in seconds

For simplicity, we'll assume $\phi = 0$.

We'll suppress the units

(L in henry's)
(R in ohms)
(I in amperes)
(V in volts)

$$\begin{cases} L\dfrac{dI}{dt} + RI = 120\cos(120\pi t) \\ \text{at } t=0,\ I=0 \text{ because} \\ \text{until } t=0, \text{ the switch was open.} \end{cases}$$

Last time: $\dfrac{dx}{dt} + \dfrac{dp}{dt}x = q$ (p, q functions of t only)

$$e^p\left(\frac{dx}{dt} + \frac{dp}{dt}x\right) = e^p q$$

$$\frac{d(e^p x)}{dt} = e^p q$$

$$\frac{dI}{dt} + \underbrace{\frac{R}{L}}_{\frac{dp}{dt}} I = \underbrace{\frac{120}{L}\cos(120\pi t)}_{q}$$

$$p = \frac{R}{L} \cdot t \Rightarrow \frac{dp}{dt} = \frac{R}{L}$$

$$\underbrace{e^{Rt/L}\left(\frac{dI}{dt} + \frac{R}{L} I\right)}_{} = e^{Rt/L} \frac{120}{L} \cos(120\pi t)$$

$$= \frac{d\left(e^{Rt/L} I\right)}{dt}$$ (check: use product rule)

$$\int d\left(e^{Rt/L} I\right) = \int e^{Rt/L} \frac{120}{L} \cos(120\pi t)\, dt$$

$$e^{Rt/L} I = \frac{120}{L} \int \left(e^{(R/L)t}\right) \cos((120\pi)t)\, dt$$

$$\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx) +$$

$$a = \frac{R}{L} \qquad b = 120\pi$$

$$e^{Rt/L} I = \frac{120}{L} \cdot \frac{e^{(R/L)t}\left(\frac{R}{L} \cos 120\pi t + 120\pi \sin 120\pi t\right)}{(R/L)^2 + (120\pi)^2}$$

$$+ c$$

$$I = \frac{120}{L} \cdot \frac{\frac{R}{L}\cos 120\pi t + 120\pi \sin 120\pi t}{(R/L)^2 + (120\pi)^2}$$

$$+ c e^{-Rt/L}$$

You can solve for c using the given fact that $I = 0$ when $t = 0$:

$$0 = \frac{120}{L} \cdot \frac{\frac{R}{L}\overbrace{\cos 0}^{1} + 120\pi \overbrace{\sin 0}^{0}}{(R/L)^2 + (120\pi)^2}$$

$$+ c \underbrace{e^{0}}_{1}$$

$$0 = \frac{120}{L} \cdot \frac{R/L}{(R/L)^2 + (120\pi)^2} + c$$

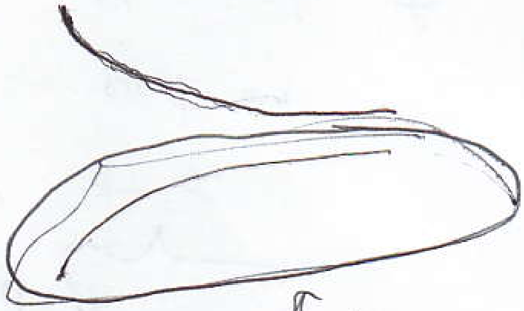
$$-\frac{120}{\cancel{L}} \cdot \frac{R/\cancel{L}}{(R/L)^2 + (120\pi)^2} = c$$

$$-\frac{120}{L^2} \cdot \frac{R}{(R/L)^2 + (120\pi)^2} = c$$

$$\frac{-120R}{R^2 + 120^2\pi^2 L^2} = c$$

$$\boxed{I = \frac{120}{L}\cdot\frac{\frac{R}{L}\cos 120\pi t + 120\pi\sin 120\pi t}{(R/L)^2 + (120\pi)^2} - \frac{120 R e^{-Rt/L}}{R^2 + 120^2\pi^2 L^2}}$$

I = oscillating + decaying



in our case, the decay term is negative

HW #1 Solve

$$\begin{cases} 3\dfrac{dx}{dt} + \dfrac{x}{10} = 120\sin(120\pi t + \pi/3) \\ x = 0 \quad \text{when } t = 0 \end{cases}$$

$$\frac{dx}{dt} + x\tan t = 3$$

$$\frac{dx}{dt} + \frac{dp}{dt}x = q \longrightarrow \frac{d(e^p x)}{dt} = e^p q$$

$$\frac{dp}{dt} = \tan t \qquad p = \ln|\sec t|$$

$$q = 3 \qquad e^p = |\sec t|$$

$$\frac{d(|\sec t|x)}{dt} = |\sec(t)|\,3$$

$$\int d(|\sec t|x) = \int |\sec t|\,3\,dt$$

$$\text{Cases}\begin{cases} \sec t > 0 \Rightarrow |\sec t| = \sec t \\ \sec t < 0 \Rightarrow |\sec t| = -\sec t \end{cases}$$

$$\begin{cases} \int d((\sec t)x) = \int (\sec t)\,3\,dt \\ \int d((-\sec t)x) = \int (-\sec t)\,3\,dt \end{cases}$$

Factor out the "–"

Get $\int d((\sec t)\, x) = \int (\sec t)\, 3\, dt$

in both cases.

$$\frac{x}{\cos t} = (\sec t)\, x = 3 \ln|\sec t + \tan t| + c$$

$$\boxed{x = 3(\cos t) \ln|\sec t + \tan t| + c \cdot \cos t}$$

HW #2 Solve

$$(\cos t) \frac{dx}{dt} + x = 1 \quad \& \quad x = 5 \text{ when } t = 0.$$