Today: Improper Integrals II (6.7)

Tomorrow: Sequences & Series (9.1, 9.2)

Dr. Wu will substitute for me.

He'll take attendance and collect homework.

I'll email my lesson plan, including HW due Monday, to you all.

Yesterday: $\int_a^\infty \frac{dx}{x^p} = \begin{cases} \text{finite}: & p > 1 \\ \text{infinite}: & p \leq 1 \end{cases}$

$a > 0$

$\int_0^\infty b^x \, dx$ $\quad b > 0$

$b^x = \left(e^{\ln b}\right)^x = e^{((\ln b) \cdot x)}$

$b^x = e^{x \ln b}$

$0 < b < 1 \Rightarrow \ln(b)$ is negative

$b = 1 \Rightarrow \ln(b)$ is 0

$1 < b \Rightarrow \ln(b)$ is positive

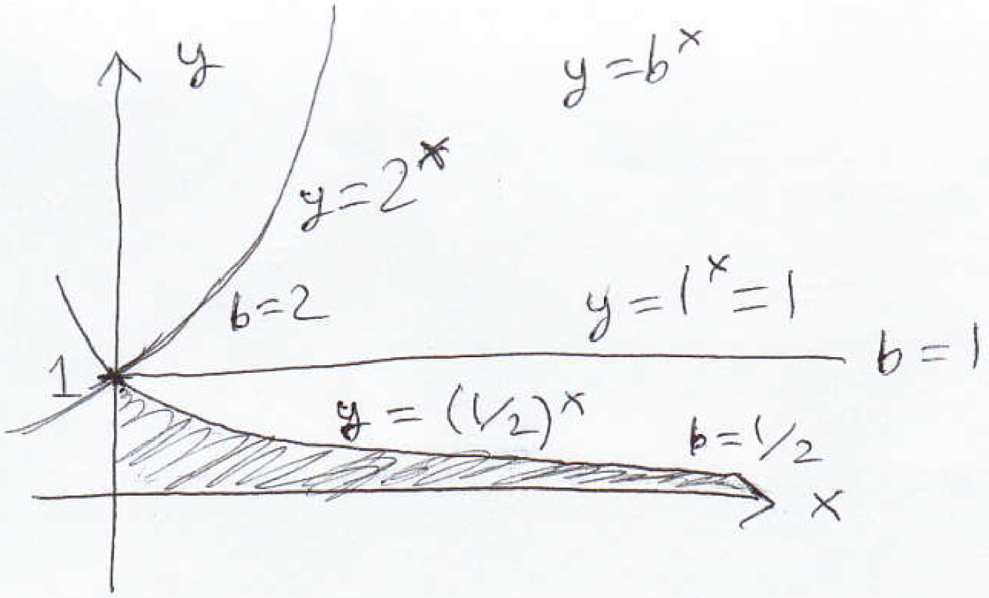
$$\int_0^\infty b^x\,dx = \int_0^\infty e^{x \ln b}\,dx = \int_0^{?} e^u\,du/\ln b$$

$$u = x \ln b \qquad 0 = 0 \ln b$$
$$du = dx \ln b$$
$$du/\ln b = dx$$

If $b=1$, then this substitution doesn't work at all: $dx = \frac{du}{\ln b} = \frac{du}{0}$.

If $0<b<1$, then $\ln b < 0$, so as $x \to \infty$, $u = x \ln b \to -\infty$.

If $b>1$, then $\ln b > 0$, so as $x \to \infty$, $u = x \ln b \to \infty$.

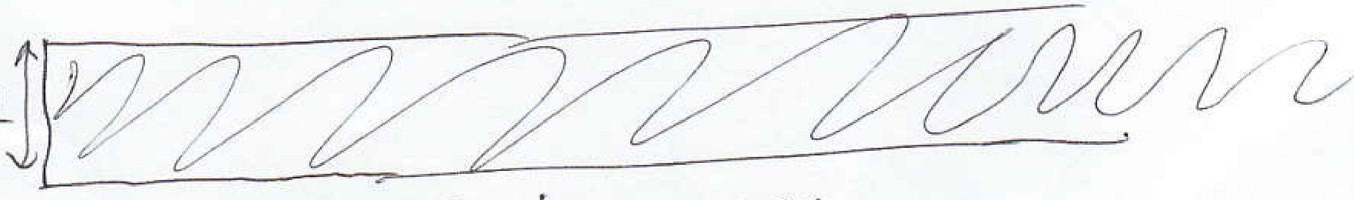


$b \ge 1 \Rightarrow \int_0^\infty b^x\,dx$ should be infinite.

$0 < b < 1 \Rightarrow \int_0^\infty b^x\,dx$ might be finite

$$b = 1 \Rightarrow \int_0^\infty \underbrace{b^x}_{1}\,dx = \lim_{c\to\infty} \int_0^c 1\,dx$$

$$= \lim_{c\to\infty} x\Big|_0^c = \lim_{c\to\infty} (c-0) = \infty$$



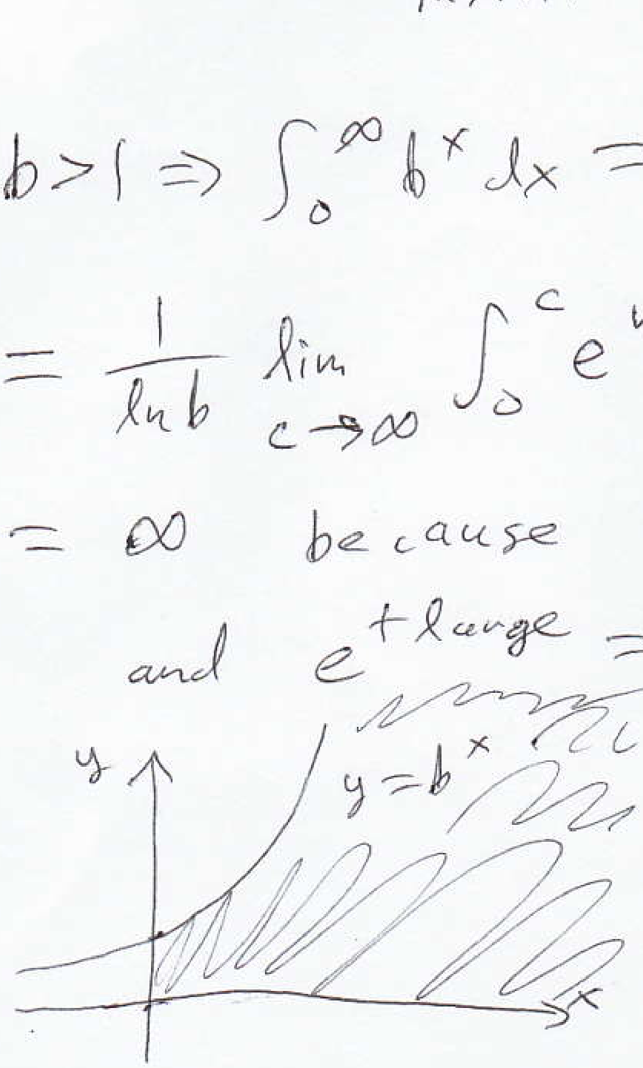
infinite area

$$b > 1 \Rightarrow \int_0^\infty b^x\,dx = \int_0^\infty e^u\,du/\underbrace{\ln b}_{>0}$$

$$= \frac{1}{\ln b} \lim_{c\to\infty} \int_0^c e^u\,du = \lim_{c\to\infty} \left(e^u\Big|_0^c\right)\overbrace{\left(\frac{1}{\ln b}\right)}^{\text{constant}}$$

$= \infty$ because $e^u\Big|_0^c = e^c - e^0 = e^c - 1$

and $e^{+\text{large}} = +\text{large}$



$0 < b < 1$: $\int_0^\infty b^x \, dx = \int_0^{-\infty} e^u \, du / \underbrace{\ln b}_{<0}$

$$= \frac{1}{\ln b} \int_0^{-\infty} e^u \, du = \frac{-1}{\ln b} \int_{-\infty}^0 e^u \, du$$

$$= \lim_{c \to -\infty} \frac{-1}{\ln b} \int_c^0 e^u \, du \qquad \int e^u du = e^u$$

$$= \lim_{c \to -\infty} \frac{-1}{\ln b} \left(e^0 - e^c\right)$$

$$= \lim_{c \to -\infty} \frac{-1}{\ln b} \left(1 - e^c\right) = \boxed{\frac{-1}{\ln b}(1 - 0)}$$

$$e^{-\text{large}} = \frac{1}{e^{+\text{large}}} = \frac{1}{+\text{large}} = +\text{small}$$

$$\int_0^\infty b^x = \begin{cases} -1/\ln b : & 0 < b < 1 \\ \infty : & 1 \le b \end{cases}$$

E.g., $\int_0^\infty \left(\frac{1}{2}\right)^x dx = \frac{-1}{\ln(1/2)} = \frac{-1}{-\ln 2} = \frac{1}{\ln 2}$

$$\int_0^\infty e^{-x} dx = \int_0^\infty (e^{-1})^x dx = \frac{-1}{\ln(e^{-1})} = \frac{-1}{-1} = 1$$

Poisson process:

Something happens at random times, ($t \geq 0$)
and the past doesn't affect
the probability distribution of
when the next thing will happen.

$0 < p < 1$ $\quad$ p = probability nothing happens in time interval $[0, 1]$

p = probability nothing happens in time interval $[t, t+1]$ (for any choice of t)

p^2 = probability nothing happens in time interval $[t, t+2]$.

$p^{\Delta t}$ = prob. nothing happens in time interval $[t, t+\Delta t]$.

$$0 < p < 1 \Rightarrow \lim_{\Delta t \to \infty} p^{\Delta t} = 0$$ (Something happens eventually.)

$\ln(p) < 0$ Let $\lambda = -\ln(p) > 0$.

$$p = e^{-\lambda}; \quad p^{\Delta t} = e^{-\lambda \Delta t}$$

Prob. 1st event happens in $[a, b]$

$$= (\text{Prob. nothing in } \underbrace{[0, a)}) \cdot (\text{Prob. something in } [a, b])$$

$$= p^a (1 - p^{b-a}) \qquad \Delta t = a - 0 = a$$

$$\Delta t = b - a$$

$$= e^{-\lambda a}(1 - e^{-\lambda(b-a)}) = e^{-\lambda a} - e^{-\lambda b}$$

$$= -e^{-\lambda t}\Big|_{t=a}^{t=b} = \int_a^b d(-e^{-\lambda t})$$

$$= \int_a^b -e^{-\lambda t} d(-\lambda t) = \int_a^b -e^{-\lambda t}(-\lambda) dt$$

Prob. 1st event $[t, t + \underbrace{dt}_{\text{small}}]$ is:

$$\int_t^{t+dt} (+e^{-\lambda t} \lambda) dt \approx +e^{-\lambda t} \lambda \, dt$$

Average waiting time (call it τ):

add up $t \times \text{Prob}(\text{1st event in } [t, t+dt])$ for all t. $(t \geq 0)$

$$\tau = \int_0^\infty t e^{-\lambda t} \lambda \, dt$$

HW: Find τ when $p = \frac{2}{3}$.