Test 6

Scores... E.g. $3/5$, $4/5$, $5/5$

is $\frac{3}{5}\cdot 35 + \frac{4}{5}\cdot 35 + \frac{5}{5}\cdot 30$

$= 3\cdot 7 + 4\cdot 7 + 5\cdot 6$

$= 21 + 28 + 30 = 79$

$$\frac{5\cancel{x^2}}{\cancel{x^2}+3} \neq \frac{5}{3}$$

$$\frac{\cancel{k^2}\cdot 3}{(\cancel{k^2}+2k+1)\cdot 2} \neq \frac{3}{(2k+1)\cdot 2}$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} \cdots$$

$$- \frac{1}{15} + \frac{1}{17} - \frac{1}{19} + \frac{1}{21} - \cdots$$

$$1 - \frac{1}{3} < \frac{\pi}{4} < 1$$

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} < \frac{\pi}{4} < 1 - \frac{1}{3} + \frac{1}{5}$$

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} < \frac{\pi}{4} < 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$$

slow convergence ...

Faster convergence:

$$\frac{1}{\sqrt{e}} = e^{-1/2} = \left(-\tfrac{1}{2}\right)^0/0! + \left(-\tfrac{1}{2}\right)^1/1! +$$

for all x: $e^x = \sum_{k=0}^{\infty} x^k/k!$

$$+ \left(-\tfrac{1}{2}\right)^2/2!$$

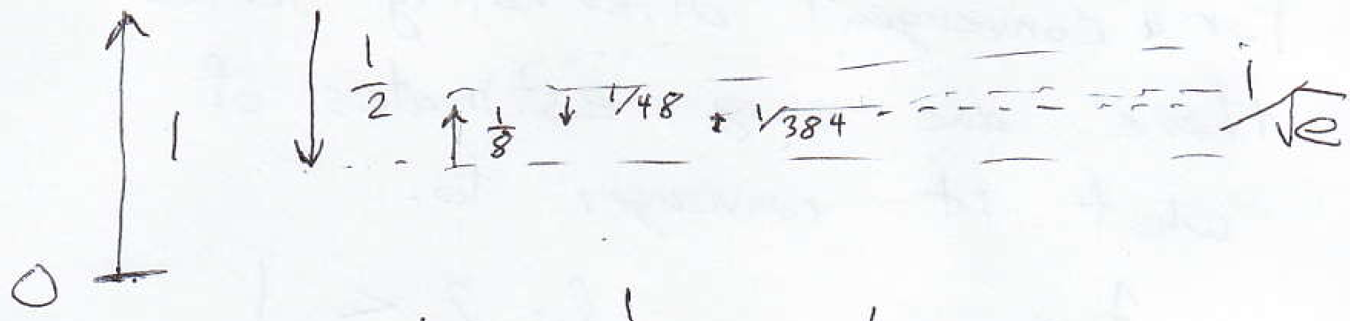
$$+ \left(-\tfrac{1}{2}\right)^3/3! + \left(-\tfrac{1}{2}\right)^4/4! + \cdots$$

$$= \frac{1}{1} - \frac{1}{2 \cdot 1} + \frac{1}{2^2 \cdot 2!} - \frac{1}{2^3 \cdot 3!} + \frac{1}{2^4 \cdot 4!} - \cdots$$

$$= 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} + \frac{1}{384} - \cdots$$

alternating between + & −

and each term is smaller than the one before it.



$$1 - \frac{1}{2} < \frac{1}{\sqrt{e}} < 1$$

0.625

$$1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} < \frac{1}{\sqrt{e}} < 1 - \frac{1}{2} + \frac{1}{8}$$

$$\frac{1}{\sqrt{e}} < 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} + \frac{1}{384}$$

≈ 0.6041

$\cancel{0.625}$

≈ 0.6067

$$\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \frac{1}{6^3} + \cdots$$

$$= \sum_{k=1}^{\infty} \frac{1}{k^3}$$

$$\sum_{k=1}^{6} \frac{1}{k^3} = \frac{1}{1^3} + \cdots + \frac{1}{6^3} = \underbrace{1.190291\ldots}_{\text{underestimate}}$$

$$\underbrace{\sum_{k=1}^{\infty} \frac{1}{k^3}}_{\text{exact value}} = \underbrace{\sum_{k=1}^{6} \frac{1}{k^3}}_{\text{under-estimate}} + \underbrace{\sum_{k=7}^{\infty} \frac{1}{k^3}}_{\text{positive}}$$

Does $\sum_{k=1}^{\infty} 1/k^3$ converge?

$$\lim_{k\to\infty} \frac{1/(k+1)^3}{1/k^3} = \lim_{k\to\infty} \frac{1}{(k+1)^3/k^3}$$

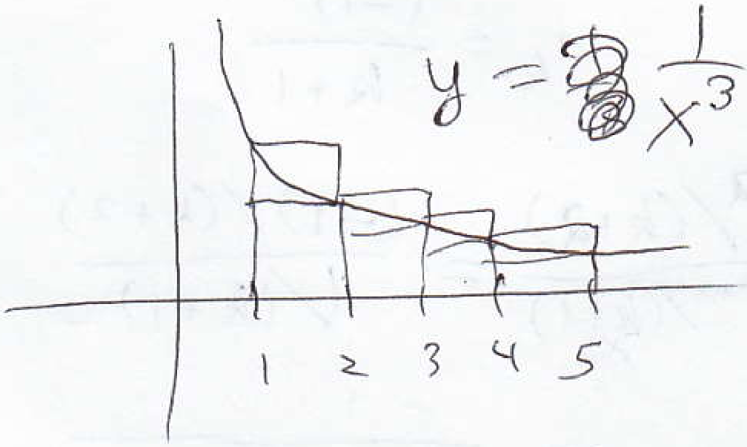
$$= \lim_{k\to\infty} \frac{1}{((k+1)/k)^3} = \lim_{k\to\infty} \frac{1}{(1+1/k)^3}$$

$$= \lim_{k\to\infty} \frac{1}{(1+0)^3} = 1$$ ratio test inconclusive...

Use integral test

when $x > 0$

$f(x) = \frac{1}{x^3}$ is positive and decreasing



Either $\int_1^\infty \frac{1}{x^3}\,dx$ & $\sum_{k=1}^{\infty} \frac{1}{k^3}$

both converge or both diverge.

$$\int_1^\infty x^{-3}\,dx = \left.\frac{x^{-2}}{-2}\right|_1^\infty \ ?$$

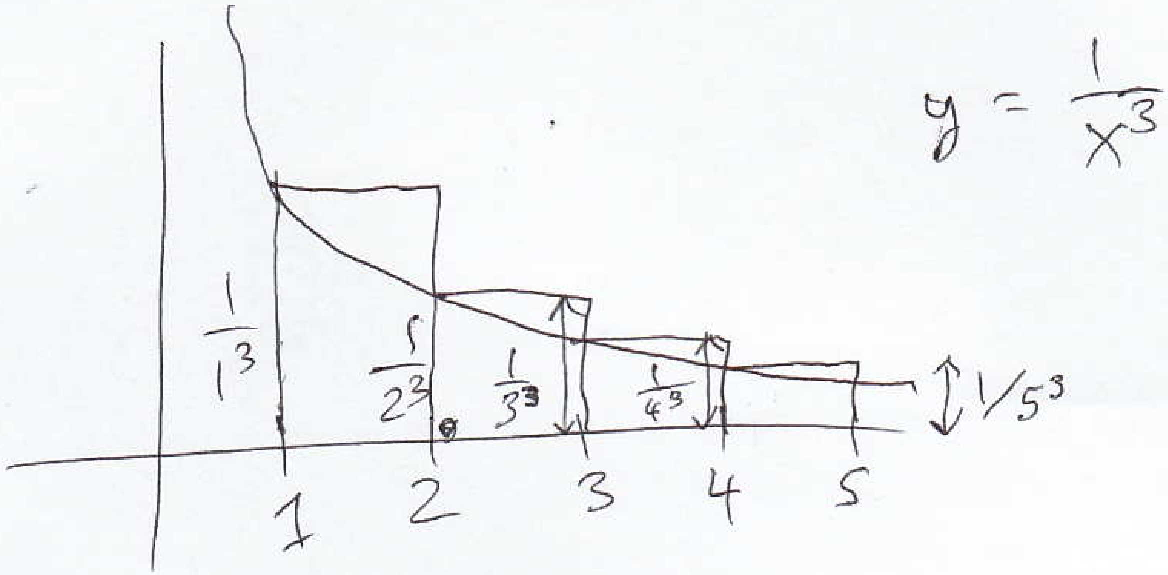
$$= \lim_{t\to\infty} \int_1^t x^{-3}\,dx$$

$$= \lim_{t\to\infty} \left.\frac{x^{-2}}{-2}\right|_1^t = \lim_{t\to\infty} \left(\frac{t^{-2}}{-2} - \frac{1^{-2}}{-2}\right)$$

$$\int_1^\infty x^{-3}\,dx = 0 - \frac{1^{-2}}{-2} = \frac{1}{2}$$

So, $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^3}$ converges

to something $> \frac{1}{2}$.



Picture of $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^3} > \int_1^\infty \frac{dx}{x^3} = \frac{1}{2}$