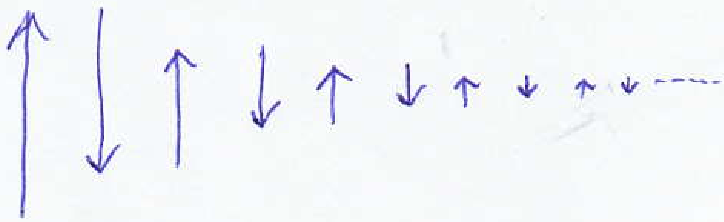
Estimating series:

If alternating and shrinking,



then upper & lower bounds are easy:

E.g. $S = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

$$\underbrace{\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots - \frac{1}{n^2}}_{\text{lower bound}} < S$$

$$S < \underbrace{\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots + \frac{1}{(n+1)^2}}_{\text{upper bound.}}$$

$$\text{spread} = \frac{1}{(n+1)^2}$$

~~If not alternating all positive~~ &

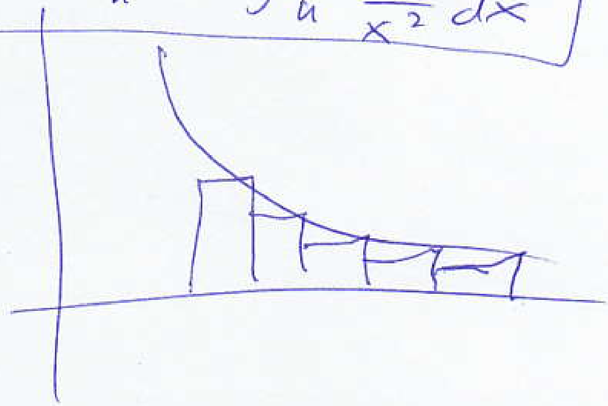
If shrinking & all positive,
then use integral estimate
if you easily extend
the sequence of terms to
a continuous function
(that is + & ↘):

$$T = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots$$

$$T = \underbrace{\boxed{\frac{1}{1^2}+\frac{1}{2^2}\cdots+\frac{1}{n^2}}}_{\text{lower bound}} + \underbrace{\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\cdots}_{< \int_n^\infty \frac{1}{x^2}\,dx}$$

upper bound is $\boxed{\frac{1}{1^2}+\cdots\frac{1}{n^2}+\int_n^\infty \frac{1}{x^2}\,dx}$

spread $= \int_n^\infty \frac{1}{x^2}\,dx \left(=\frac{1}{n}\right)$



If there isn't an integral estimate available, then you do a geometric series estimate

$$U = \frac{3}{4} + \frac{3^2}{4^2 \cdot 2} + \frac{3^3}{4^3 \cdot 3} + \frac{3^4}{4^4 \cdot 4} + \frac{3^5}{4^5 \cdot 5} + \cdots$$

$$\cancel{\frac{3}{4}\left(1 + \frac{3}{4 \cdot 2} + \frac{3}{4 \cdot 3} + \frac{3}{4 \cdot 4}\right)}$$

$$= \underbrace{\frac{3}{4} + \cdots + \frac{3^n}{4^n \cdot n}}_{\text{lower bound}} + \underbrace{\frac{3^{n+1}}{4^{n+1}(n+1)} + \frac{3^{n+2}}{4^{n+2}(n+2)} + \cdots}$$

$$= \binom{\text{lower}}{\text{bound}} + \frac{3^{n+1}}{4^{n+1}(n+1)}\left[1 + \frac{3(n+1)}{4(n+2)} + \frac{3^2(n+1)}{4^2(n+3)} + \cdots\right]$$

$$< \binom{\text{lower}}{\text{bound}} + \frac{3^{n+1}}{4^{n+1}(n+1)}\left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \cdots\right]$$

$$= \underbrace{(\text{lower bound}) + \frac{3^{n+1}}{4^{n+1}(n+1)} \cdot \frac{1}{1-(3/4)}}_{\text{upper bound}}$$