Another Formula for R_n:

$$\frac{\overbrace{\int_0^x \frac{(x-t)^n}{n!} f^{(n+1)}(t)\,dt}^{R_n}}{\int_0^x \frac{(x-t)^n}{n!}\,dt} = \frac{((x-\xi)^n/n!)\,f^{(n+1)}(\xi)}{((x-\xi)^n/n!)} = f^{(n+1)}(\xi)$$

because if $A(x) = \int_0^x \frac{(x-t)^n}{n!} f^{(n+1)}(t)\,dt$

and $B(x) = \int_0^x \frac{(x-t)^n}{n!}\,dt$, then

$$\frac{A(x)}{B(x)} = \frac{A(x)-A(0)}{B(x)-B(0)} \underset{\uparrow}{=} \frac{A'(\xi)}{B'(\xi)} \underset{\uparrow}{=} \frac{((x-\xi)^n/n!)\,f^{(n+1)}(\xi)}{((x-\xi)^n/n!)}$$

Generalized Mean Value Theorem (section 9.10)

1st Fundamental Theorem of Calculus

for some ξ strictly between 0 and x.

So, $A(x)/B(x) = f^{(n+1)}(\xi)$.

$u = x-t$
$du = -dt$
$dt = -du$

$$B(x) = \int_0^x (x-t)^n\,dt/n! = \int_x^0 u^n(-du)/n! = \int_0^x \frac{u^n}{n!}\,du$$

$$= \frac{u^{n+1}}{n!(n+1)}\Big|_0^x = \frac{x^{n+1}}{(n+1)!} - 0$$

So, $R_n = A(x) = B(x) f^{(n+1)}(\xi) = \boxed{f^{(n+1)}(\xi)\frac{x^{n+1}}{(n+1)!}}$