MATH 2415 Final Exam Name: _____

1. Given vectors $\vec{u} = \langle 3, 4, 8 \rangle$ and $\vec{w} = \langle 7, 2, 1 \rangle$, there is a unique pair of vectors $\vec{a}$ and $\vec{b}$ for which the vector $\vec{a}$ is parallel to $\vec{u}$, the vector $\vec{b}$ is perpendicular to $\vec{u}$, and $\vec{a} + \vec{b} = \vec{w}$. Find $\vec{a}$ and $\vec{b}$.

2. What is the radius of curvature of the parametrized ellipse $\vec{r}(t) = \langle 7\cos(t), 1, 2\sin(t)\rangle$ at $t = \pi/3$?

3. Find $\iint_P y^2\,dA$ where P is the parallelogram $ABCD$ with vertices $A = (0,0)$, $B = (7,0)$, $C = (8,2)$, $D = (1,2)$.

4. Given $\vec{F} = \langle x^2 + 7y, y^2 + 2z, z^2 + x \rangle$, find $\operatorname{curl} \vec{F}$ and $\operatorname{div} \vec{F}$ at $(3, 4, 8)$.

5. Given $\vec{F} = \langle x^2 + 7y, y^2 + 2z, z^2 + x \rangle$ again, find the flux of $\operatorname{curl} \vec{F}$ through the surface

$$\Sigma = \{(x, y, z) \mid z = r^2 \leq 7^2\}.$$