MATH 2415 FINAL EXAM

Name: _____

Testing conditions:

- 3 hour time limit;
- notes, books, and calculators are allowed;
- inter-student communication, telecommunication, and internet access are not allowed.

Date: May 13, 2014.

1. **[20 points]** Find the angle between the planes

$$H = \{(x, y, z) : 2y + 3z = -4\} \text{ and } K = \{(x, y, z) : 2x + 5z = y\}.$$

2. [30 points] Consider the surface S determined by the equation $x^3 + 2y^3 + 3z^3 = 10xyz$.

(a) Find an equation for the plane tangent to S at $(1, 2, 1)$.

(b) Use this tangent plane to find an approximate value of the z near 1 for which $(1.01, 1.99, z)$ is on S.

3. **[20 points]** Find the average y coordinate of the interior of the counterclockwise loop $(x, y) = (t^3 - t, 1 - t^2)$, $-1 \leq t \leq 1$.

4. [20 points] Find the flux of $\langle xy, xz, x - y\rangle$ through the boundary of the tetrahedron $T = \{(x, y, z) : 0 \leq x \leq 2y \leq 4z \leq 12\}$.

5. [10 points] Let $\mathbf{F} = \langle yz^3, xz^2, 2xyz \rangle$. Prove that $\int_L \mathbf{F} \cdot d\mathbf{r} = 0$ for every circular loop L contained in the plane $P = \{(x, y, z) : x = 5\}$.