MATH 2415 FINAL EXAMINATION

Name: ________________________________

Date: May 13, 2013.

Exercise	Point Possible	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1. [25 points] Compute the flux of the field $\langle xy^2z, yx^2z, x+y\rangle$ through the boundary of the cylindrical region $\{(x,y,z) : x^2+y^2 \leq 5 \text{ and } 2 \leq z \leq 4\}$.

2. [25 points] The ellipsoid surface $E = \{(x, y, z) : (x/5)^2 + (y/6)^2 + (z/7)^2 = 1\}$ can be parametrized using "stretched" spherical coordinates:

$$
\begin{aligned}
x &= 5 \sin\phi \cos\theta \\
y &= 6 \sin\phi \sin\theta \\
z &= 7 \cos\phi \\
0 &\leq \phi \leq \pi \\
0 &\leq \theta \leq 2\pi
\end{aligned}
$$

Find a formula for the surface area differential $d\sigma$ of E. The only variables in your formula should be ϕ, θ, $d\phi$, and $d\theta$.

3. [25 points] Use the fact that curl $\langle bz, cx, ay\rangle = \langle a, b, c\rangle$ (assuming $\langle a, b, c\rangle$ is constant) to compute the flux of the constant vector field $\langle 1, 3, 5\rangle$ through the upper hemisphere $H = \{(x, y, z) : x^2 + y^2 + z^2 = 1 \text{ and } z \geq 0\}$.

4. [25 points] Compute the three angles of the triangle with vertices $(0, 0, 1)$, $(1, 2, 3)$, and $(-4, 1, 2)$.