HW5

① If $|\vec{u}|=4$, $\angle(\vec{u},\vec{v})=\pi/5$,
and $|\vec{u}\times\vec{v}|=8$, then $|\vec{v}|=?$

② Find some unit vectors $\vec{u},\vec{v},\vec{w}$
such that $(\vec{u}\times\vec{v})\times\vec{w}\neq\vec{u}\times(\vec{v}\times\vec{w})$.

③ $(\vec{i}+2\vec{j})\times(\vec{k}-3\vec{i})=?$

④ If $|\vec{u}|=5$, then what is
$\left(\left(\vec{u}+(\vec{u}\times\vec{v})\right)\cdot\vec{u}\right)-\left(3\vec{v}\cdot(\vec{u}\times\vec{v})\right)$?

⑤ Find a unit vector perpendicular
to both $\langle 1,2,0\rangle$ and $\langle 0,5,-1\rangle$.

HW 5

① $8 = |\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin \angle(\vec{u}, \vec{v})$

$= 4|\vec{v}| \sin \frac{\pi}{5} \Rightarrow |\vec{v}| = \frac{2}{\sin(\pi/5)} \approx 3.4026$

② $(\vec{i} \times \vec{i}) \times \vec{j} = \vec{0} \times \vec{j} = \vec{0}$

$\vec{i} \times (\vec{i} \times \vec{j}) = \vec{i} \times \vec{k} = -\vec{j} \neq \vec{0}$

(There are many other examples.)

③ $(\vec{i} + 2\vec{j}) \times (\vec{k} - 3\vec{i})$ (or $\langle 1, 2, 0 \rangle \times \langle -3, 0, 1 \rangle$)

$= \vec{i} \times \vec{k} - 3\vec{i} \times \vec{i} + 2\vec{j} \times \vec{k} - 6\vec{j} \times \vec{i}$

$= -\vec{j} \qquad + 2\vec{i} \qquad + 6\vec{k}$

$= \langle 2, -1, 6 \rangle$

(4) $\vec{u}, \vec{v} \perp \vec{u} \times \vec{v} \Rightarrow \vec{u} \cdot (\vec{u} \times \vec{v}) = 0 = \vec{v} \cdot (\vec{u} \times \vec{v})$.

So, $((\vec{u} + (\vec{u} \times \vec{v})) \cdot \vec{u}) - (3\vec{v} \cdot (\vec{u} \times \vec{v}))$

$$= \vec{u} \cdot \vec{u} + 0 - 0 = |\vec{u}|^2 = \boxed{25}$$

(5) $\langle 1, 2, 0 \rangle \times \langle 0, 5, -1 \rangle$

$= \langle -2, 1, 5 \rangle \perp \langle 1, 2, 0 \rangle, \langle 0, 5, -1 \rangle,$

but $\langle -2, 1, 5 \rangle$ is not a unit vector.

$\boxed{\frac{1}{\sqrt{30}} \langle -2, 1, 5 \rangle}$ works.

(So does $\frac{-1}{\sqrt{30}} \langle -2, 1, 5 \rangle$.)

HW 6

Let $A = (0, 1, 0)$; $B = (5, 5, 5)$;
$C = (-1, 2, 3)$; $D = (5, 0, 2)$.

① Express $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{BC}$, $\overrightarrow{BD}$, and $\overrightarrow{CD}$ using $\langle\ ,\ ,\ \rangle$ form.

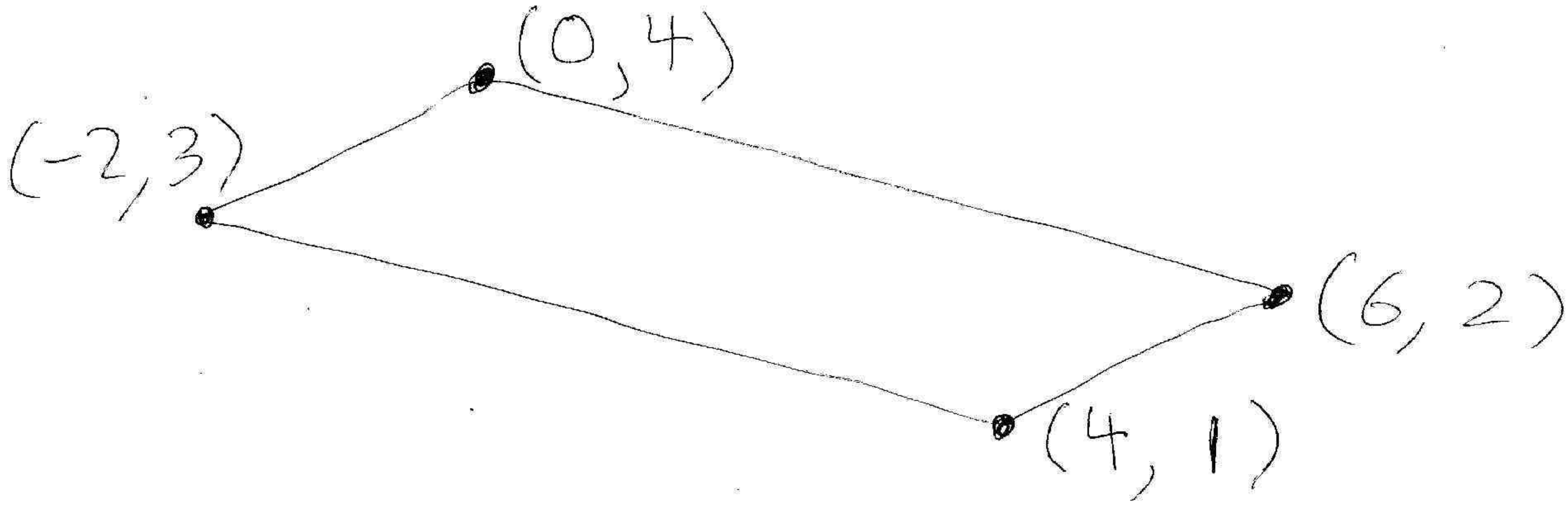
② Find the areas of the triangles ABC, ABD, ACD, and BCD.

③ Add these to find the surface area of tetrahedron ABCD.

④ Find the volume of ABCD.

⑤ Find the total length of ABCD's 6 edges.

(6) Find the area of this parallelogram.



HW6

(1) $\overrightarrow{AB} = \langle 5, 4, 5 \rangle$; $\overrightarrow{AC} = \langle -1, 1, 3 \rangle$;

$\overrightarrow{AD} = \langle 5, -1, 2 \rangle$; $\overrightarrow{BD} = \langle 0, -5, -3 \rangle$;

$\overrightarrow{BC} = \langle -6, -3, -2 \rangle$; $\overrightarrow{CD} = \langle 6, -2, -1 \rangle$

(2) $|\triangle ABC| = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{\sqrt{530}}{2} \approx 11.51$

$|\triangle ABD| = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AD}| = \frac{\sqrt{1019}}{2} \approx 15.96$

$|\triangle ACD| = \frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{AD}| = \frac{\sqrt{330}}{2} \approx 9.083$

$|\triangle BCD| = \frac{1}{2}|\overrightarrow{BC} \times \overrightarrow{BD}| = 35/2 = 17.5$

(3) ≈ 54.05

(4) $\frac{1}{6}|(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD}| = \frac{73}{6} \approx 12.17$

(5) $|\overrightarrow{AB}| + |\overrightarrow{AC}| + \cdots = \sqrt{54} + \sqrt{11} + \cdots \approx 36.15$

(6) $A=(-2,3,0)$, $B=(4,1,0)$,

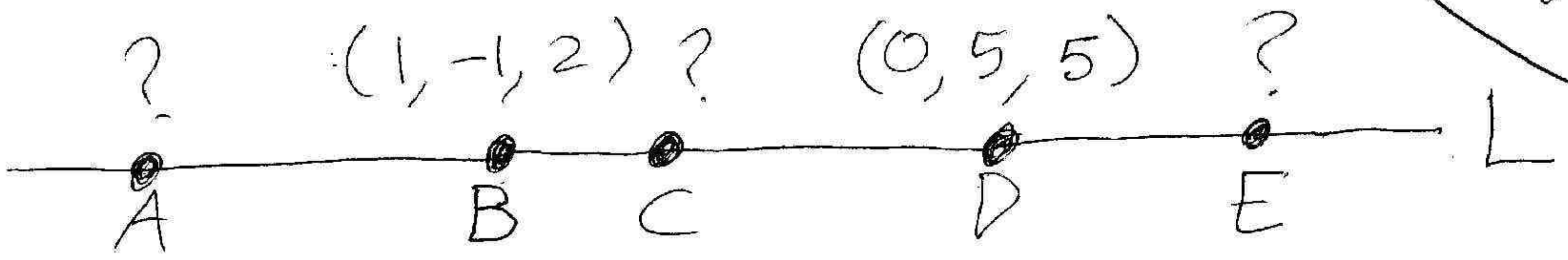
$C=(0,4,0)$, $D=(6,2,0)$.

$$\text{area}\underbrace{(ABCD)}_{\text{parallelogram}} = |\vec{AB} \times \vec{AC}| = 10$$

Note: It really is a parallelogram because $\vec{AB} = \vec{CD}$ & $\vec{AC} = \vec{BD}$.

Day 7 HW

(1)



Parametrize the line L through B & D.

Then find points A, C, E on L

such that B is between A & C,

C is between B & D,

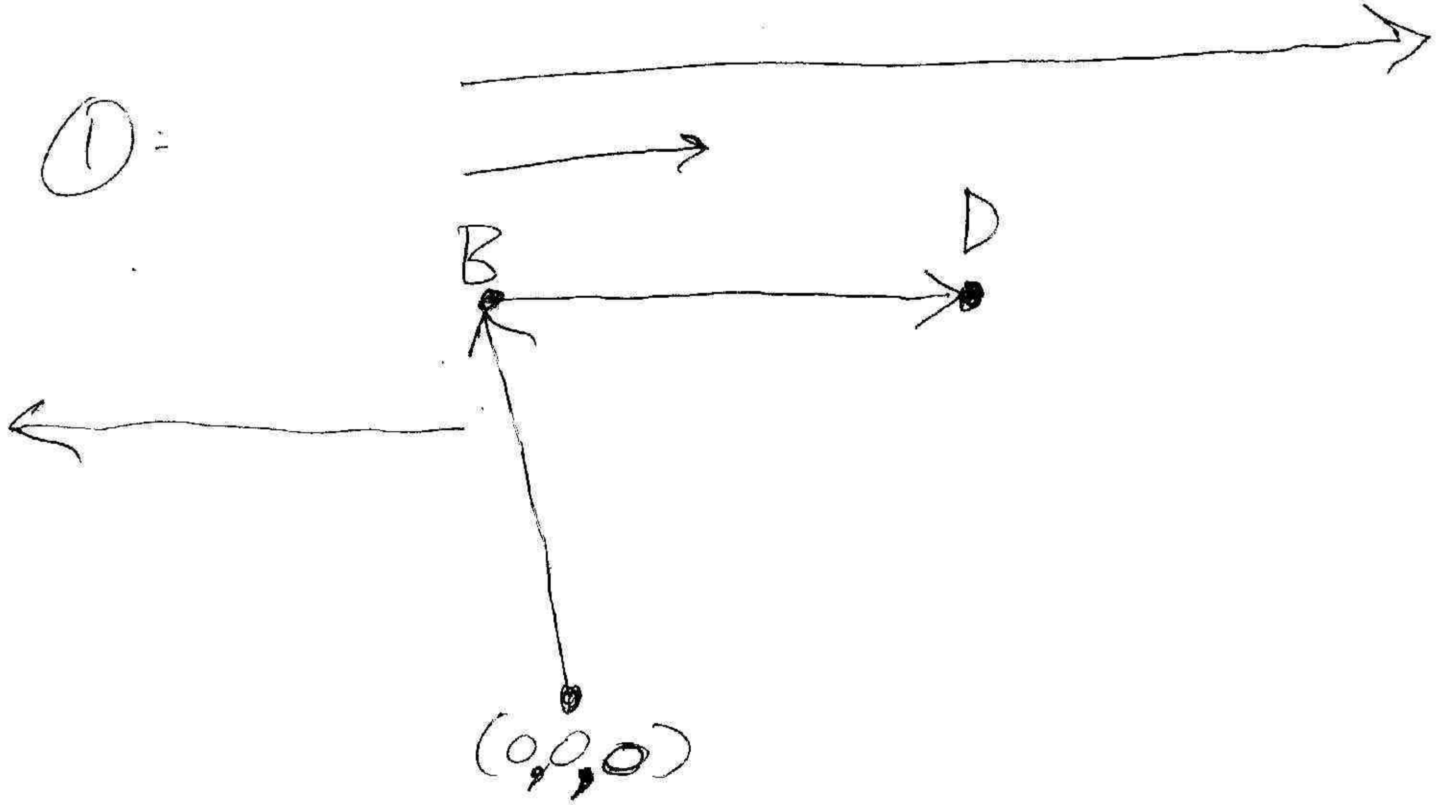
and D is between C & E.

(2) Parametrize the line through (0, 0, 1) that is perpendicular to $\langle 1,1,1\rangle$ & $\langle 1,1,2\rangle$.

(3) Find a unit vector parallel to the line satisfying $x-2y+3z=4$ & $5x+6z=7$.

④ Find Cartesian equations for the line in ①.

Hint for ①:



HW 7

(1) $\vec{r} = \overrightarrow{OB} + t\,\overrightarrow{BD}$ parametrizes L:

$$\boxed{\langle x, y, z\rangle = \langle 1, -1, 2\rangle + t\langle -1, 6, 3\rangle}$$

(or: $x = 1 - t$, $y = -1 + 6t$, $z = 2 + 3t$)

Any choice of $t < 0$ gives an acceptable A.

For example: $\overrightarrow{OA} = \langle 1, -1, 2\rangle + (-1)\langle -1, 6, 3\rangle$

$$\Rightarrow \boxed{A = (2, -7, -1)}$$

Any choice of t with $0 < t < 1$ works for C.

For example: $\overrightarrow{OC} = \langle 1, -1, 2\rangle + \frac{1}{2}\langle -1, 6, 3\rangle$

$$\Rightarrow \boxed{C = (1/2, 2, 7/2)}$$

Any choice of $t > 1$ works for E.

$t = 2$, for example, yields $\boxed{E = (-1, 11, 8).}$

(2) Call the line L. $(0,0,1) \in L \parallel \langle 1,-1,0 \rangle$

because $\langle 1,-1,0 \rangle = \langle 1,1,1 \rangle \times \langle 1,1,2 \rangle$ is perpendicular to both $\langle 1,1,1 \rangle$ & $\langle 1,1,2 \rangle$.

So, $\boxed{\langle x,y,z \rangle = \langle 0,0,1 \rangle + t\langle 1,-1,0 \rangle}$

parametrizes L.

(3) Choosing $x=0$, then $z=0$, I find two points on the line: $A = (0, -1/4, +7/6)$, then $B = (+7/5, -13/10, 0)$.

why? $\begin{cases} x - 2y + 3z = 4 \\ 5x \quad + 6z = 7 \\ x \qquad = 0 \end{cases} \Rightarrow \begin{cases} y = -1/4 \\ z = 7/6 \\ x = 0 \end{cases}$; $\begin{cases} x - 2y + 3z = 4 \\ 5x \quad + 6z = 7 \\ \qquad z = 0 \end{cases} \Rightarrow \begin{cases} y = -13/10 \\ x = 7/5 \\ z = 0 \end{cases}$

Parallel to the line is $\overrightarrow{AB} = \langle 7/5, -21/20, -7/6 \rangle$ $= \frac{7}{60}\langle 12, -9, -10 \rangle$. A unit vector parallel to the line is $\boxed{\dfrac{\langle 12, -9, -10 \rangle}{\sqrt{12^2 + 9^2 + 10^2}}}$

$\approx \langle 0.666, -0.499, -0.555 \rangle$.

(The opposite arrow $\approx \langle -.666, +.499, +.555 \rangle$ is the only other correct solution.)

(4) $x = 1 - t \Longrightarrow t = 1 - x$

$y = -1 + 6t \Longrightarrow$ $\boxed{y = -1 + 6(1-x)}$

$z = 2 + 3t \Longrightarrow$ $\boxed{z = 2 + 3(1-x)}$

Alternative solution to (3): The line is the intersection of two planes $\perp$ to $\langle 1, -2, 3 \rangle$ & $\langle 5, 0, 6 \rangle$, respectively. So, the line is $\parallel$ to their cross product $\langle -12, +9, +10 \rangle$ & to $\frac{\langle -12, +9, +10 \rangle}{\sqrt{12^2+9^2+10^2}}$.

HW8

1. Parametrize the plane containing $(1,0,1)$, $(2,3,4)$, and $(-1,5,-6)$.

2. Give a Cartesian equation for the same plane as 1.

3. We can rearrange $x+y+z=5$ as $0 = 1(x-0) + 1(y-0) + 1(z-5)$ to see that $x+y+z=5$ describes the plane containing $(0,0,5)$ & $\perp$ to $\langle 1,1,1 \rangle$.

Use an analogous trick to find a vector perpendicular to the plane $3x-4z=1$.

HW 8

① $\vec{r} = \overrightarrow{OA} + s\,\overrightarrow{AB} + t\,\overrightarrow{AC}$

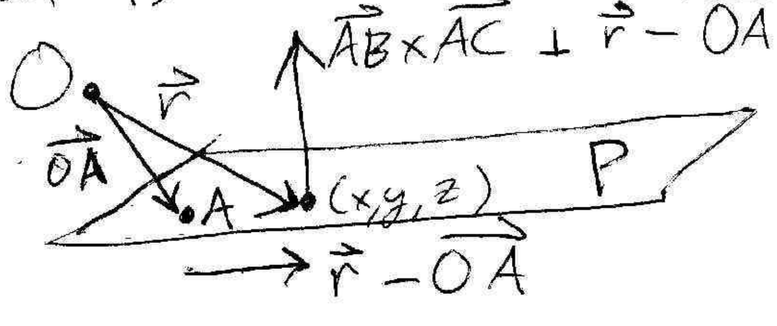
where
$A = (1,0,1)$
$B = (2,3,4)$
$C = (-1,5,-6)$
$O = (0,0,0)$

$$\boxed{\begin{aligned} x &= 1 + s - 2t \\ y &= \phantom{1+{}} 3s + 5t \\ z &= 1 + 3s - 7t \end{aligned}}$$

② The plane P through ABC is parallel to $\overrightarrow{AB}$ & $\overrightarrow{AC}$; hence, it is perpendicular to $\overrightarrow{AB} \times \overrightarrow{AC} = \langle -36, 1, 11 \rangle$

So, P is described by $0 = (\vec{r} - \overrightarrow{OA}) \cdot \langle -36, 1, 11 \rangle$, which is more usefully written as:

$$\boxed{0 = -36(x-1) + y + 11(z-1)}$$



(3) $3x-4z=1$

$$\Leftrightarrow 3x-4z-1=0$$

$$\Leftrightarrow 3(x-0)+0(y-0)+(-4)\left(z+\frac{1}{4}\right)=0$$

$$\Leftrightarrow \langle 3,0,-4\rangle \cdot \left(\langle x,y,z\rangle - \langle 0,0,\tfrac{-1}{4}\rangle\right)=0.$$

So, $3x-4z=1$ describes the plane through $(0,0,-1/4)$ that is perpendicular to $\boxed{\langle 3,0,-4\rangle}$.

(You can just read the $3,0,-4$ from the coefficients of $3x-4z=1$!)