HW 9

1. Find the distance from $(4,0,8)$ to the line $L = \{(x,y,z) \mid x+y+z=1 \ \& \ z-y=2\}$.

2. Find the distance from $(5,5,5)$ to the plane containing the points $(1,2,0)$ & $(0,2,3)$ and parallel to the line L from 1.

Hint: start by finding two arrows parallel to the plane.

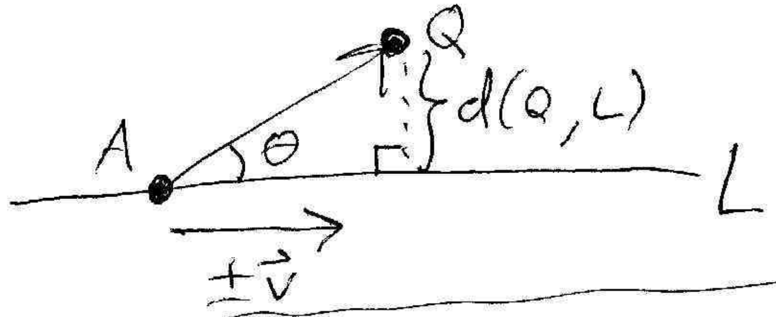
$Q=(4,0,8)$ "$\therefore$" means "Therefore," HW 9

① $A=(x,0,z)\in L \Rightarrow z=2 \Rightarrow x=-1$. $\therefore A=(-1,0,2)$

$B=(x,y,0)\in L \Rightarrow y=-2 \Rightarrow x=3$. $\therefore B=(3,-2,0)$

$\vec{v}=\overrightarrow{AB}=\langle 4,-2,-2\rangle \parallel L$

$\overrightarrow{AQ}=\langle 5,0,6\rangle$



$d(Q,L)=|\overrightarrow{AQ}|\sin\theta$

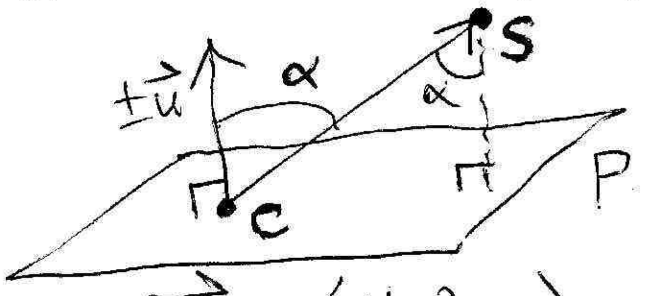
$=|\overrightarrow{AQ}\times\vec{v}|/|\vec{v}| = \boxed{\sqrt{(12^2+34^2+10^2)/(4^2+2^2+2^2)}}$

② $S=(5,5,5)$; $C=(1,2,0)$; $D=(0,2,3)$; $\overrightarrow{CD}=\langle -1,0,3\rangle$

$C, D\in$ plane $P \parallel \vec{v}$. $\therefore \overrightarrow{CD}, \vec{v} \parallel P$ $\therefore P \perp \overrightarrow{CD}\times\vec{v}$

$\vec{u}=\overrightarrow{CD}\times\vec{v}=\langle 6,10,2\rangle$. $d(S,P)=|\overrightarrow{CS}|\cos\alpha$

$\Downarrow$



$\overrightarrow{CS}=\langle 4,3,0\rangle$

$d(S,P)=|\overrightarrow{CS}\cdot\vec{u}|/|\vec{u}|$

$=|24+30+0|/\sqrt{6^2+10^2+2^2}$

$=\boxed{27/\sqrt{35}}$

HW10

(1) If $(x, y, z) = (-3, -4, -5)$, then $(r, \theta, z) = ?$

(2) If $(r, \theta, z) = (7, -5\pi/4, 1)$, then $(x, y, z) = ?$

(3) Describe (geometrically) the following sets of points.

$F = \{(x, y, z) : r + z = 5\}$

$G = \{(x, y, z) : 0 \leq \theta \leq \pi/2 \ \& \ r \leq 5 \ \& \ |z| \leq 4\}$

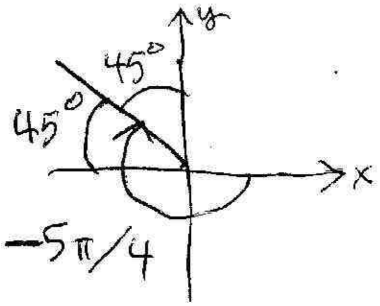
$H = \{(x, y, z) : z = r\cos\theta\}$

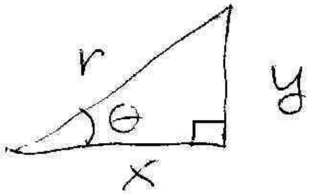
$K = \{(x, y, z) : z = r \ \& \ \theta = \pi/3\}$

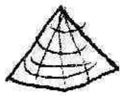
HW10

① $(r,\theta,z) = (\underset{\sqrt{(-3)^2+(-4)^2}}{\underset{\uparrow}{5}}, \cos^{-1}(\underset{x/r}{\underset{\uparrow}{\tfrac{-3}{5}}}), -5) \approx (5, 2.214, -5)$

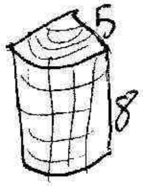
② $(x,y,z) = (\underbrace{-\frac{7}{\sqrt{2}}}_{7\cos\frac{-5\pi}{4}}, \underbrace{+\frac{7}{\sqrt{2}}}_{7\sin\frac{-5\pi}{4}}, 1) \approx (-4.950, 4.950, 1)$



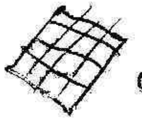


③ 

- $z = 5 - r$ describes a downward opening cone with vertex $(x,y,z) = (0,0,5)$. F



- G is a quarter sector of a solid cylinder whose axis is the z-axis, radius is 5, and height is 8.



- H is the plane $z = x$.

- K is a ray on an upward opening cone $(z=r)$; the ray starts at the origin.

HW 11

1. Convert $(x, y, z) = (3, -4, 2)$ to spherical coordinates (ρ, φ, θ).

2. Convert $(r, \theta, z) = (5, -\pi/6, 4)$ to spherical coordinates.

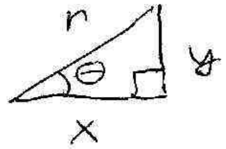
3. Describe the surface $\{(x, y, z) : \rho = 5\}$.

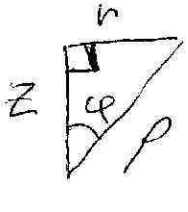
4. Convert $(\rho, \varphi, \theta) = (11, 5\pi/6, -\pi/2)$ to Cartesian coordinates (x, y, z)

5. Convert $(\rho, \varphi, \theta) = (7, \pi/3, \pi/4)$ to cylindrical coordinates (r, θ, z).

6. Describe the solid region $\{(x, y, z) : \rho \le 3 \ \& \ 0 \le \theta \le \pi/2 \ \& \ 0 \le \varphi \le \pi/2\}$

HW11





① $$(\rho,\varphi,\theta)=\Big(\overbrace{\sqrt{29}}^{\sqrt{3^2+(-4)^2+2^2}},\ \cos^{-1}\overbrace{\frac{2}{\sqrt{29}}}^{z/\rho},\ \cos^{-1}\overbrace{\frac{3}{5}}^{\frac{x}{r}=x/\sqrt{x^2+y^2}}\Big)$$
$$\approx(5.385,\ 1.190,\ .9273)$$

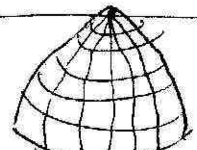
② $$(\rho,\varphi,\theta)=\Big(\overbrace{\sqrt{41}}^{\sqrt{r^2+z^2}},\ \cos^{-1}\frac{4}{\sqrt{41}},\ -\frac{\pi}{6}\Big)$$
$$\approx(6.403,\ .8961,\ -.5236)$$

③ A (hollow) sphere of radius 5 & center (0,0,0).

④ $$(x,y,z)=\Big(11\underbrace{\sin\frac{5\pi}{6}}_{1/2}\underbrace{\cos\frac{-\pi}{2}}_{0},\ 11\underbrace{\sin\frac{5\pi}{6}}_{1/2}\underbrace{\sin\frac{-\pi}{2}}_{-1},\ 11\underbrace{\cos\frac{5\pi}{6}}_{-\sqrt{3}/2}\Big)$$
$$\approx(5.5,\ 0,\ -9.526)$$

⑤ $$(r,\theta,z)=\Big(7\sin\frac{\pi}{3},\ \frac{\pi}{4},\ 7\cos\frac{\pi}{3}\Big)\approx(6.062,\ .7854,\ 3.5)$$

⑥ An eighth of a solid ball.



HW12

(1) Let $(\rho, \varphi, \theta) = (t, \pi/6, 5t)$

(This is a spiral on a cone.)

Convert this to $\vec{r} = \langle x, y, z \rangle = \langle ?, ?, ? \rangle$.

Find $\vec{r}$, $\vec{v}$, $|\vec{v}|$, $\vec{T}$ at $t = \pi/20$.

HW 12

$$\underbrace{\vec{r}}_{\langle x, y, z\rangle} = \underbrace{(t)}_{\rho}\left\langle \overbrace{\sin\frac{\pi}{6}}^{1/2}\cos 5t,\ \overbrace{\sin\frac{\pi}{6}}^{1/2}\sin 5t,\ \underbrace{\cos\frac{\pi}{6}}_{\sqrt{3}/2}\right\rangle$$

$$\vec{v} = (1)\left\langle \frac{1}{2}\cos 5t,\ \frac{1}{2}\sin 5t,\ \frac{\sqrt{3}}{2}\right\rangle +$$

$$+(t)\left\langle \frac{5}{2}(-\sin 5t),\ \frac{5}{2}\cos 5t,\ 0\right\rangle$$

At $t = \frac{\pi}{20}$, $\vec{r} = \frac{\pi}{20}\left\langle \frac{1}{2}\cos\frac{\pi}{4},\ \frac{1}{2}\sin\frac{\pi}{4};\ \frac{\sqrt{3}}{2}\right\rangle$

$$= \frac{\pi}{40}\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ \sqrt{3}\right\rangle \text{ and}$$

$$\vec{v} = \frac{1}{2}\left[\left\langle \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ \sqrt{3}\right\rangle + \frac{5\pi}{40}\left\langle -\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}},\ 0\right\rangle\right]$$

Numerically, at $t = \pi/20$,

$\vec{r} \approx \langle .0555, .0555, .1360 \rangle$,

$\vec{v} \approx \langle .0759, .6312, .8660 \rangle$,

$|\vec{v}| \approx 1.074$, and

$\vec{T} = \frac{\vec{v}}{|\vec{v}|} \approx \langle .0706, .5876, .8061 \rangle$