HW13

① $\int_0^1 \sqrt{(3t^2-1)^2+(4t^3-1)^2+(5t^4-1)^2}\,dt$

$\approx \boxed{1.64111}$

② $\vec{r} = \langle 5\cos 3t, 5\sin 3t, 2t\rangle$

$\vec{v} = \frac{d\vec{r}}{dt} = \langle -15\sin 3t, 15\cos 3t, 2\rangle$

$|\vec{v}| = \sqrt{15^2(\sin^2 3t + \cos^2 3t) + 2^2} = \sqrt{15^2+2^2} = \sqrt{229}$

$\int_0^{4\pi} |\vec{v}|\,dt = \boxed{4\pi\sqrt{229}}$

HW14

(1) $\cos \angle(\vec{v}, \vec{a}) = \dfrac{\vec{v}\cdot\vec{a}}{|\vec{v}||\vec{a}|} = \dfrac{1}{3}$;

so, $0 < \angle(\vec{v}, \vec{a}) < \pi/2 = 90°$;

so, the motion is turning and speeding up.

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(2) $\vec{v} = (-e^{-t})\langle -\sin(e^{-t}), \cos(e^{-t})\rangle$

$\vec{a} = (e^{-t})\langle -\sin(e^{-t}), \cos(e^{-t})\rangle$

$\quad + (e^{-2t})\langle -\cos(e^{-t}), -\sin(e^{-t})\rangle$

Using $\vec{r} = \langle \cos\theta, \sin\theta\rangle$ where $\theta = e^{-t}$, $\dfrac{d\theta}{dt} = -e^{-t}$, $\dfrac{d^2\theta}{dt^2} = e^{-t}$

At $t=1$:

$$\boxed{\begin{aligned} \vec{r} &\approx \langle .9331, .3596\rangle \\ \vec{v} &\approx \langle .1323, -.3433\rangle \\ \vec{a} &\approx \langle -.2586, .2946\rangle \end{aligned}}$$

We use the numerical values of $\vec{r}, \vec{v}, \vec{a}$ @ $t=1$ to complete the rest of the exercise.

HW 14

$$|\vec{v}| = \sqrt{(.132303\ldots)^2 + (-.343265\ldots)^2} \approx \boxed{.3679}$$

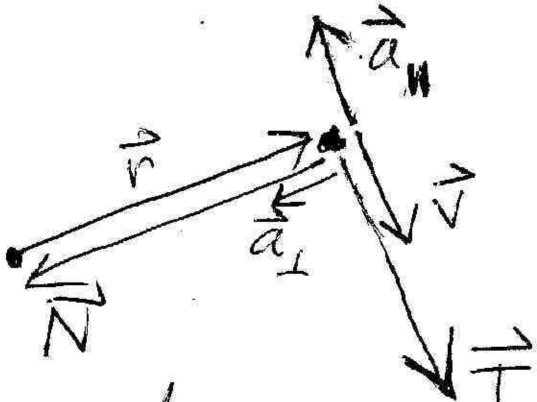
$$\vec{T} = \frac{1}{|\vec{v}|}\vec{v} = \left\langle \frac{.132303\ldots}{.367879\ldots}, \frac{-.343265\ldots}{.367879\ldots} \right\rangle \approx \boxed{\langle .3596, -.9331 \rangle}$$

(Use a few extra significant figures in inputs to formulas so that the outputs will have the desired accuracy.)

$$\frac{d|\vec{v}|}{dt} = \vec{T} \cdot \vec{a} \approx \boxed{-.3679}; \quad \vec{a}_{\parallel} = (\vec{T} \cdot \vec{a})\vec{T} \approx \boxed{\langle -.1323, .3433 \rangle}$$

$$\vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} \approx \boxed{\langle -.1263, -.04867 \rangle}$$

$$\vec{N} = \frac{1}{|\vec{a}_{\perp}|}\vec{a}_{\perp} \approx \boxed{\langle -.9331, -.3596 \rangle}$$



(Some numbers appeared in multiple answers because $(e^{-t})' = -e^{-t}$ & the curve is special: position is moving along the unit circle $\{(x,y) \mid x^2+y^2=1\}$.)

HW 15

① $\vec{v} = \vec{r}\,' = \langle -t^{-2}, -2t^{-3}, -3t^{-4} \rangle = \langle -1, -2, -3 \rangle$

$\vec{a} = \vec{v}\,' = \langle 2t^{-3}, 6t^{-4}, 12t^{-5} \rangle = \langle 2, 6, 12 \rangle$

$R = |\vec{v}|^3 / |\vec{v} \times \vec{a}| = \sqrt{14}^{\,3} / \sqrt{76} \approx 6.00877$

② $\vec{r} = \langle t, \sin t, 0 \rangle$

$\vec{v} = \langle 1, \cos t, 0 \rangle = \langle 1, 0, 0 \rangle$ @ $x = t = \pi/2$

$\vec{a} = \langle 0, -\sin t, 0 \rangle = \langle 0, -1, 0 \rangle$ @ $x = t = \pi/2$

$R = |\vec{v}|^3 / |\vec{v} \times \vec{a}| = 1$

③ $\vec{v} = \vec{r}\,' = \langle -AB\sin(Bt), AB\cos(Bt), C \rangle$

$\vec{a} = \vec{v}\,' = -AB^2 \langle \cos(Bt), \sin(Bt), 0 \rangle$

$|\vec{v}|^2 = A^2B^2(\sin^2 Bt + \cos^2 Bt) + C^2 = A^2B^2 + C^2$

$\vec{v} \times \vec{a} = (-AB^2) \langle -C\sin(Bt), C\cos(Bt), -AB(\sin^2 Bt + \cos^2 Bt) \rangle$

$|\vec{v} \times \vec{a}|^2 = (AB^2)^2 (C^2(\sin^2 Bt + \cos^2 Bt) + (AB)^2)$

$$|\vec{v}\times\vec{a}|^2 = (AB^2)^2(C^2 + A^2B^2) = A^2B^4(A^2B^2 + C^2)$$

HW 15

$$R = \frac{|\vec{v}|^3}{|\vec{v}\times\vec{a}|} = \sqrt{\frac{(A^2B^2+C^2)^3}{A^2B^4(A^2B^2+C^2)}} = \boxed{\frac{A^2B^2+C^2}{AB^2}}$$

HW16

① $\vec{r}_0 = \frac{8\pi}{3}\left\langle \cos\frac{8\pi}{3}, \sin\frac{8\pi}{3}\right\rangle = \left\langle -\frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}}\right\rangle$

$\vec{v} = \langle \cos t, \sin t\rangle + t\langle -\sin t, \cos t\rangle$ $\quad \Rightarrow \vec{r}_0 \approx \langle -4.19, 7.26\rangle$

$\vec{v}_0 = \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2}\right\rangle + \left\langle -\frac{4\pi}{\sqrt{3}}, -\frac{4\pi}{3}\right\rangle \approx \langle -7.76, -3.32\rangle$

$\vec{a} = \langle -\sin t, \cos t\rangle + \langle -\sin t, \cos t\rangle + t\langle -\cos t, -\sin t\rangle$

$\vec{a}_0 = \langle -\sqrt{3}, -1\rangle + \left\langle \frac{4\pi}{3}, -\frac{4\pi}{\sqrt{3}}\right\rangle \approx \langle 2.46, -8.26\rangle$

$\vec{T}_0 = \frac{\vec{v}_0}{|\vec{v}_0|} \approx \langle -.919, -.394\rangle$

$\vec{a}_\perp = \vec{a} - (\vec{T}\cdot\vec{a})\vec{T} \approx \langle 3.37, -7.86\rangle$ @ $t = t_0$

HW 16

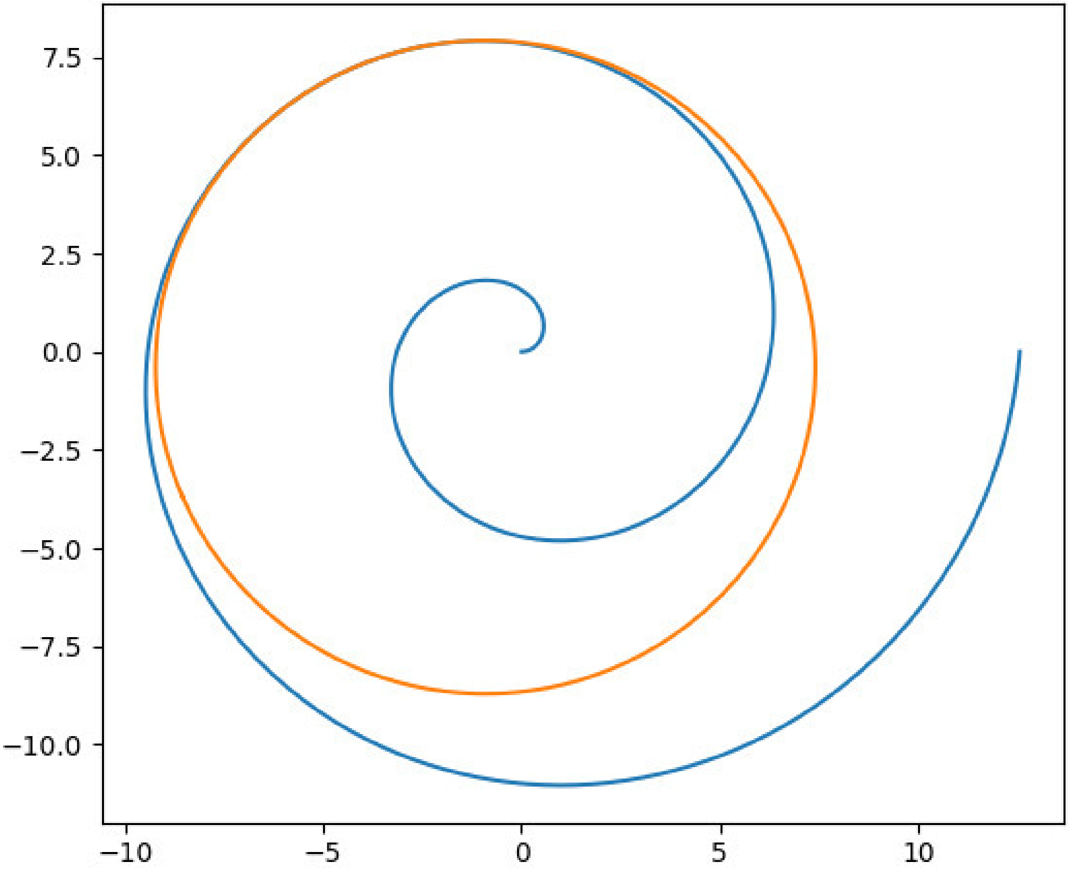
$$\vec{N} = \frac{\vec{a}_\perp}{|\vec{a}_\perp|} \Rightarrow \vec{N}_0 \approx \langle .394, -.919 \rangle$$

$$R = \frac{|\vec{v}|^2}{|\vec{a}_\perp|} \Rightarrow R_0 \approx 8.32 \Rightarrow \begin{cases} R_0 \vec{T}_0 \approx \langle -7.65, -3.28 \rangle \\ R_0 \vec{N}_0 \approx \langle 3.28, -7.65 \rangle \end{cases}$$

$$\vec{r}_{osc} = \vec{r}_0 + R\vec{T}_0 \cos\alpha + R\vec{N}_0(\sin\alpha + 1); \quad 0 \le \alpha \le 2\pi$$

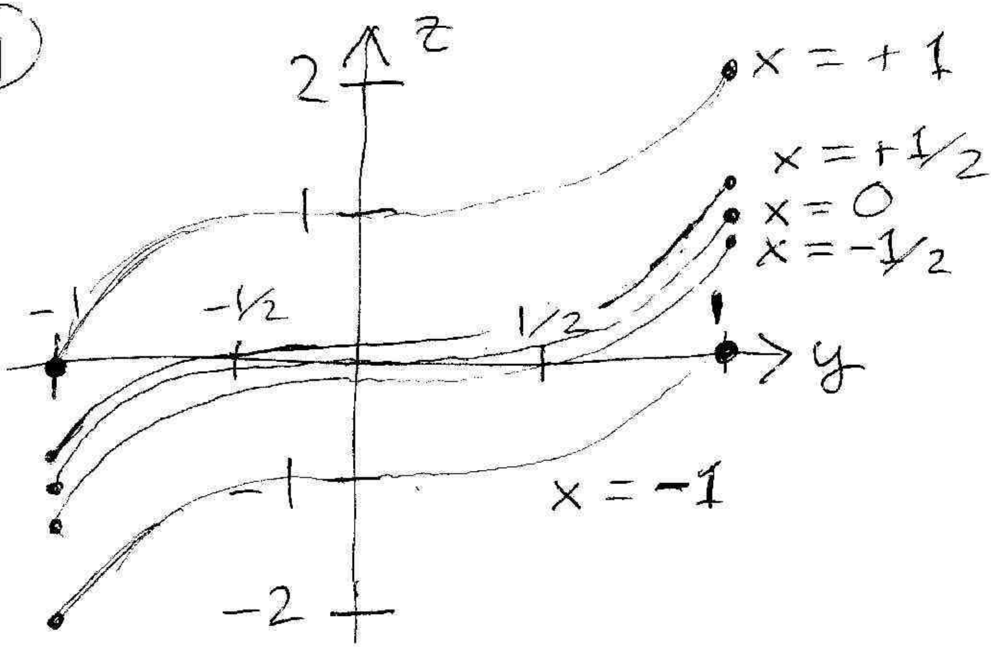
$$\boxed{\begin{aligned} x &\approx -4.19 - 7.65\cos(\alpha) + 3.28(\sin(\alpha) + 1) \\ y &\approx 7.26 - 3.28\cos(\alpha) - 7.65(\sin(\alpha) + 1) \\ 0 &\le \alpha \le 2\pi \end{aligned}}$$

(See attached plot.)


7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10
−5
0
5
10

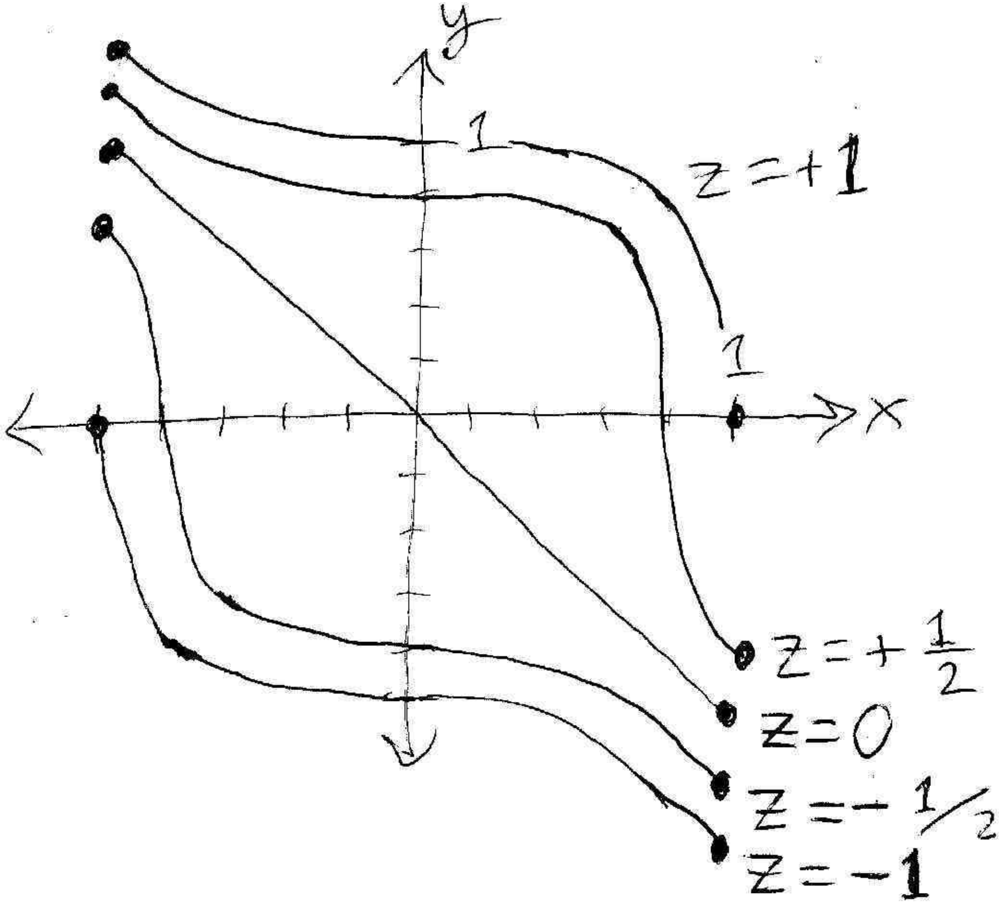
HW18

1



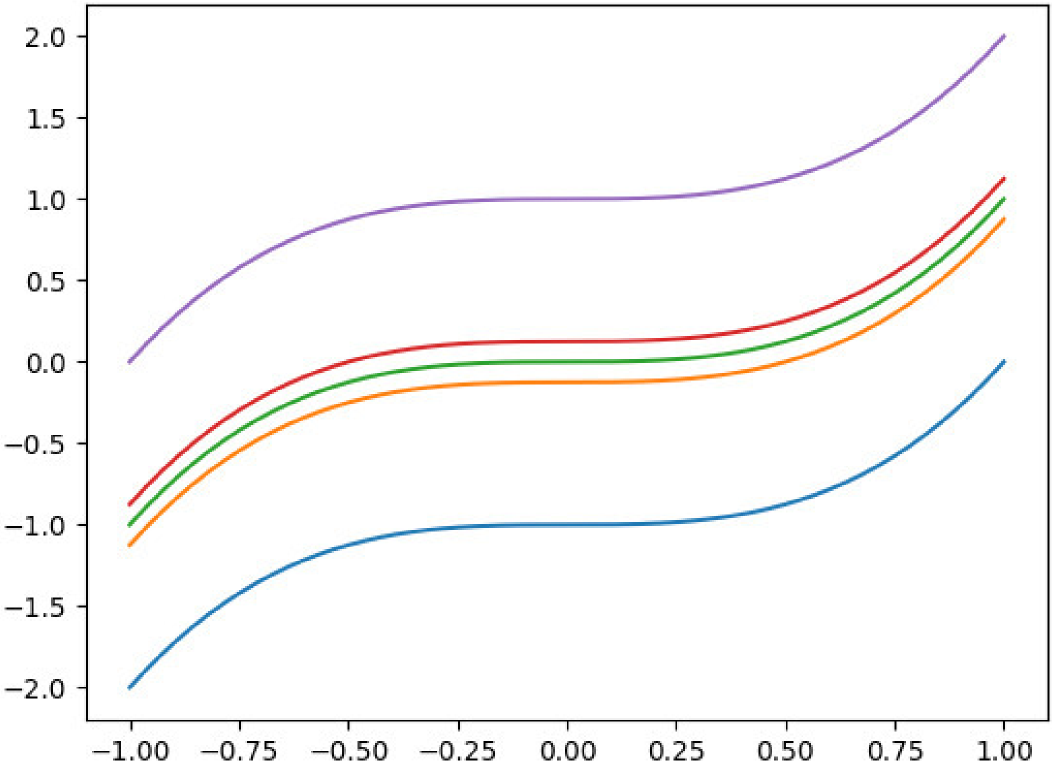
2: Identical to 1 except x replaces y and y replaces x in the labels.

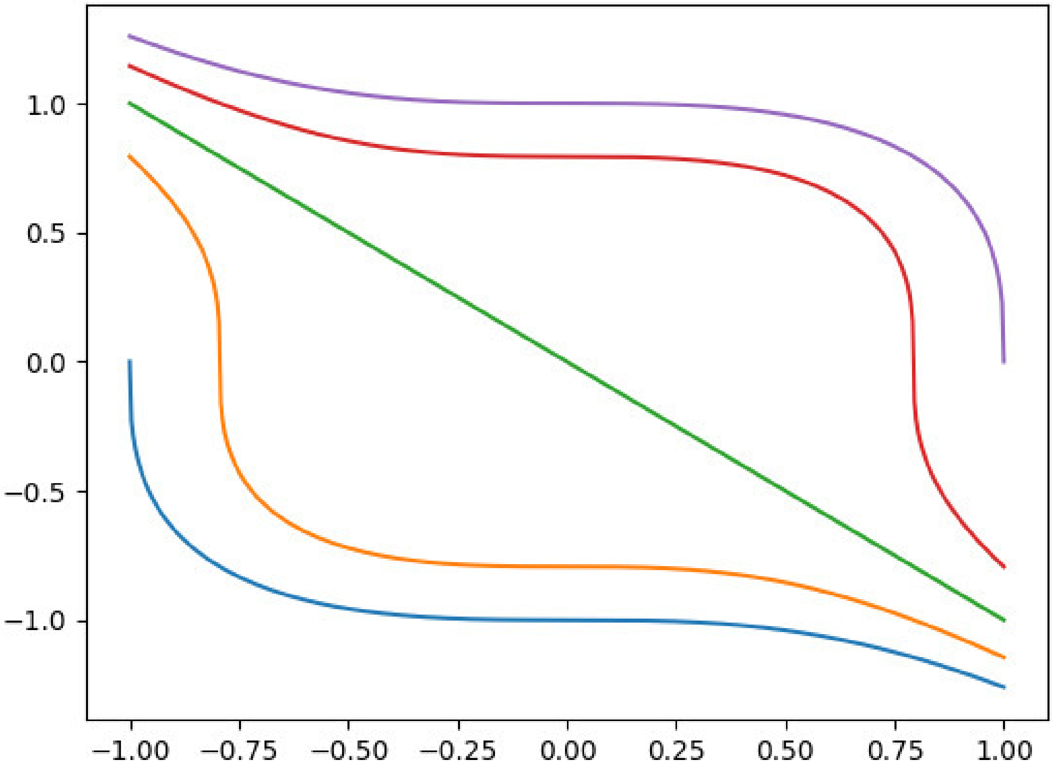
3



For 3, solve $z = x^3 + y^3$ for y:

$$y = \sqrt[3]{z - x^3}.$$


2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


1.0
0.5
0.0
−0.5
−1.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00