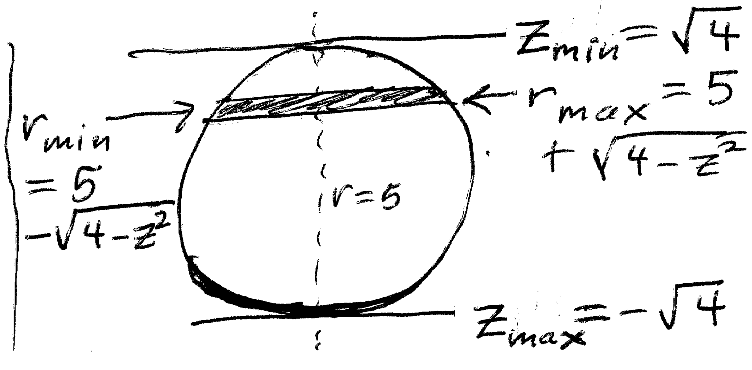
HW 40

① $I_y = \iiint_H (x^2+z^2)\,dm$ & $I_z = \iiint_H (x^2+y^2)\,dm$

$x^2+z^2 = r^2\cos^2\theta + z^2$ & $dm = 3\,dV = 3\,r\,dr\,d\theta\,dz$

& $x^2+y^2 = r^2$ (in cylindrical coordinates).

$$\iiint_H = \int_{z=-2}^{z=2}\int_{r=5-\sqrt{4-z^2}}^{r=5+\sqrt{4-z^2}}\int_{\theta=0}^{\theta=2\pi}$$

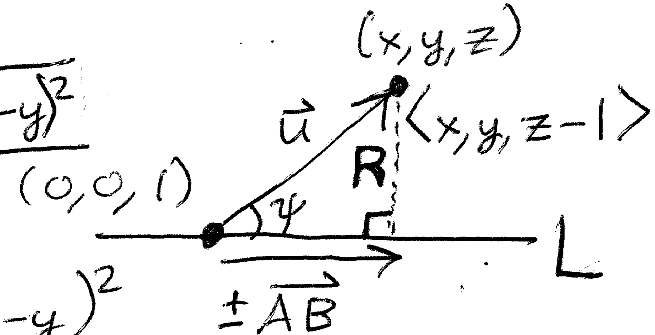


$I_y \approx \boxed{17765}$ & $I_z \approx \boxed{33162}$.

② Let R = (distance from (x,y,z) to L).

Choosing $y=0$, we get $A=(0,0,1)\in L$. $\Rightarrow \overrightarrow{AB} = \langle 1,3,0\rangle \parallel L$

Choosing $y=1$, we get $B=(1,3,1)\in L$.



$$R = |\vec{u}|\sin\psi = \frac{|\vec{u}\times\overrightarrow{AB}|}{|\overrightarrow{AB}|} = \frac{\sqrt{(3(1-z))^2+(z-1)^2+(3x-y)^2}}{\sqrt{1^2+3^2+0^2}}$$

$$R^2 = \frac{1}{10}\left(10(z-1)^2+(3x-y)^2\right) = (z-1)^2 + \frac{1}{10}(3x-y)^2$$

$dm = x\,dV = x\,dz\,dy\,dx$

$$\iiint_H = \int_{x=0}^{x=2}\int_{y=0}^{y=2-x}\int_{z=0}^{z=2-x-y} \quad ; \quad I = \iiint_H R^2\,dm = \boxed{\frac{38}{9}}$$

① $\langle x,y\rangle = \overrightarrow{OA} + u\overrightarrow{AB} + v\overrightarrow{AC} = \langle 1+2u+6v,\ 1+5u+2v\rangle$ | HW 41

$dA_{▰} = |\overrightarrow{AB}\times\overrightarrow{AC}|\,du\,dv = |\langle 0,0,-26\rangle|\,du\,dv = 26\,du\,dv$

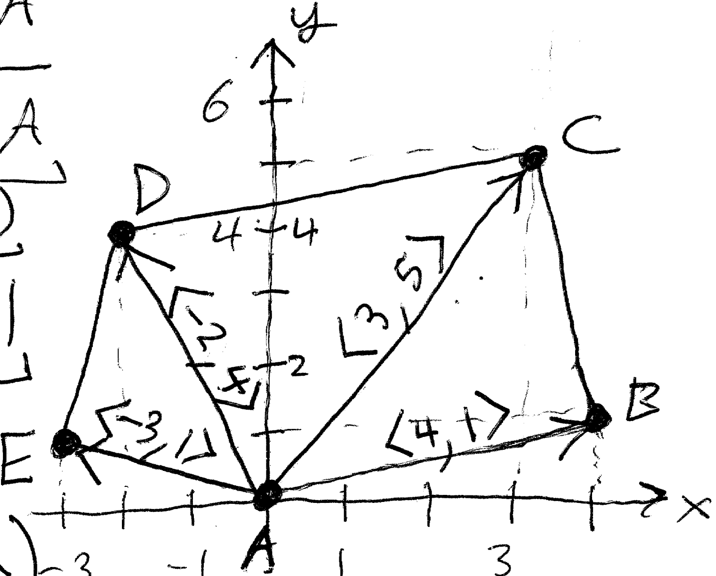
(add z-component 0) $\left(dA_{▰} \neq dA_{\blacksquare} = dx\,dy\right)$

$$\iint_{ABC} \frac{dA_{\blacksquare}}{xy} = \int_0^1\left(\int_0^{1-u} \frac{26}{(1+2u+6v)(1+5u+2v)}\,dv\right)du$$

$\boxed{\approx 1.467}$ (There is no exact formula.)

② Compute the average of x^2 inside ABCDE

as
$$\frac{\iint_{ABC} x^2\,dA + \iint_{ACD} x^2\,dA + \iint_{ADE} x^2\,dA}{\underbrace{\iint_{ABC} 1\,dA}_{\substack{\text{area}(ABC)\\ \underbrace{\tfrac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|}_{17/2}}} + \underbrace{\iint_{ACD} 1\,dA}_{\substack{\text{area}(ACD)\\ \underbrace{\tfrac{1}{2}|\overrightarrow{AC}\times\overrightarrow{AD}|}_{11}}} + \underbrace{\iint_{ADE} 1\,dA}_{\substack{\text{area}(ADE)\\ \underbrace{\tfrac{1}{2}|\overrightarrow{AD}\times\overrightarrow{AE}|}_{5}}}}$$



(By $\langle a,b\rangle\times\langle c,d\rangle$ I mean $\langle a,b,0\rangle\times\langle c,d,0\rangle$.)

To find $\iint_\Delta x^2\,dA$ for each of the three Δ's, | HW 41

use $\langle x,y\rangle = \overrightarrow{OA} + u\overrightarrow{AF} + v\overrightarrow{AG}$ & $dA = |\overrightarrow{AF}\times\overrightarrow{AG}|\,du\,dv$

for each (F, G) in $\{(B,C),\ (C,D),\ (D,E)\}$.

Then $\iint_\Delta = \int_{u=0}^{u=1}\int_{v=0}^{v=1-u}$ for each Δ.

$$\iint_{ABC} x^2\,dA = \int_0^1\left(\int_0^{1-u}(4u+3v)^2(17)\,dv\right)du = 629/12$$

$$\iint_{ACD} x^2\,dA = \int_0^1\left(\int_0^{1-u}(3u-2v)^2(22)\,dv\right)du = 77/6$$

$$\iint_{ADE} x^2\,dA = \int_0^1\left(\int_0^{1-u}(-2u-3v)^2(10)\,dv\right)du = 95/6$$

All together, $$\text{average}(x^2) = \frac{\frac{629}{12}+\frac{77}{6}+\frac{95}{6}}{\frac{17}{2}+11+5} = \frac{\frac{973}{12}}{\frac{49}{2}} = \boxed{\frac{139}{42}}$$

$$\langle x, y\rangle = \overrightarrow{OA} + u\overrightarrow{AB} + v\overrightarrow{AC} = \langle 4-6u+v,\ 3+2u-4v\rangle$$

$$dA_{\blacksquare} = |\langle -6, 1, 0\rangle \times \langle 2, -4, 0\rangle|\, du\,dv = 22\, du\,dv$$

$\boxed{\text{HW } 42}$

$$\iint_{ABDC} = \int_{u=0}^{u=1}\int_{v=0}^{v=1} \ ; \qquad r = \sqrt{(4-6u+v)^2 + (3+2u-4v)^2}$$

$$\iint_{ABDC} r\, \underbrace{dx\,dy}_{dA_{\blacksquare}} = \int_0^1\left(\int_0^1 \sqrt{(4-6u+v)^2+(3+2u-4v)^2}\, dv\right) \underbrace{du \cdot 22}_{dA_{\blacksquare}} \approx \boxed{69.39}$$

$$\vec{G} = \langle -y, x\rangle, \quad \vec{H} = \langle x, x+y\rangle, \quad \vec{I} = \langle x, -y\rangle,$$

$$\vec{J} = \langle y, -1\rangle, \quad \vec{K} = \langle x^2, y^2\rangle, \quad \vec{L} = \langle 2, x\rangle$$

$\boxed{\text{HW } 43}$

Key ideas: $\langle +, +\rangle \nearrow$, $\langle +, 0\rangle \rightarrow$, $\langle +, -\rangle \searrow$,

$\langle -, +\rangle \nwarrow$, $\langle -, 0\rangle \leftarrow$, $\langle -, -\rangle \swarrow$,

$\langle 0, +\rangle \uparrow$, $\langle 0, 0\rangle \bullet$, $\langle 0, -\rangle \downarrow$.

$x<0$, $y>0$	$x>0$, $y>0$
$x<0$, $y<0$	$x>0$, $y<0$

HW 44

① $\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \langle t^2, \frac{1}{t}, (t^3)^2 \rangle \cdot \langle 1, 2t, 3t^2 \rangle \, dt$

$= \int_0^1 (t^2 + 2 + 3t^8)\, dt \boxed{= \frac{1}{3} + 2 + \frac{3}{9} = \frac{8}{3}}$

② $\int_C \vec{F}\cdot d\vec{r} = \int_0^{-\infty} \Big(e^{3t} \langle -\sin^3 t, \cos^3 t \rangle \cdot e^t \langle \cos t - \sin t, \sin t + \cos t \rangle \, dt \Big)$

$= \int_0^{-\infty} e^{4t} (-\cos t \sin^3 t + \sin^4 t + \cos^3 t \sin t + \cos^4 t)\, dt$

$= \frac{1}{4} \int_0^{-\infty} e^{4t} \big(-(\sin 2t)(1-\cos 2t) + (1-\cos 2t)^2 + (\sin 2t)(1+\cos 2t) + (1+\cos 2t)^2 \big)\, dt = \frac{1}{4}\int_0^{-\infty} e^{4t}(2 + 2(\cos 2t)(\sin 2t) + 2\cos^2 2t)\, dt$

$= \frac{1}{4} \int_0^{-\infty} e^{4t} (2 + \sin 4t + (1 + \cos 4t))\, dt = \frac{1}{16} \int_0^{-\infty} e^u (3 + \sin u + \cos u)\, du$

$= \boxed{-3/16}$ because $\int_0^{-\infty} e^u \, du = e^u \Big|_0^{-\infty} = -1$ &

$J = \int_0^{-\infty} e^u (\sin u + \cos u)\, du \underset{IBP}{=} \overbrace{e^u (\sin u + \cos u) \Big|_0^{-\infty}}^{-1} - \int_0^{-\infty} e^u (\cos u - \sin u)\, du$

$\underset{IBP}{=} -1 - \underbrace{\left(e^u (\cos u - \sin u) \Big|_0^{-\infty} \right)}_{-1} + \int_0^{-\infty} e^u (-\sin u - \cos u)\, du = -J$

$\Rightarrow J = -J \Rightarrow J = 0.$