$$\frac{dx}{dt} = 4 \;\&\; \frac{dr}{dt} = 7 \;\&\; \frac{d\theta}{dt} = ?$$

@ $(r, \theta) = (5, \pi/3)$

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\begin{tikzcd}
 & r \arrow[r, "\frac{\partial x}{\partial r}"] \arrow[ddr, "\frac{\partial y}{\partial r}"'] & x \\
t \arrow[ur, "\frac{dr}{dt}"] \arrow[dr, "\frac{d\theta}{dt}"'] & & \\
 & \theta \arrow[r, "\frac{\partial y}{\partial \theta}"'] \arrow[uur, "\frac{\partial x}{\partial \theta}"] & y
\end{tikzcd}$$

$$\star\ \underbrace{\frac{dx}{dt}}_{4} = \frac{\partial x}{\partial r}\underbrace{\frac{dr}{dt}}_{7} + \frac{\partial x}{\partial \theta}\underbrace{\frac{d\theta}{dt}}_{\text{solve for this}}$$

$$1\cos\underbrace{\theta}_{\pi/3} \qquad \underbrace{r}_{5}(-\sin\underbrace{\theta}_{\pi/3})$$

$$4 = \underbrace{\left(\cos\frac{\pi}{3}\right)}_{1/2} 7 + (-5)\underbrace{\left(\sin\frac{\pi}{3}\right)}_{\sqrt{3}/2}\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{4 - 7\cos\pi/3}{-5\sin\pi/3}\left(= \frac{7/2 - 4}{5\sqrt{3}/2} = \frac{7-8}{5\sqrt{3}} = \frac{-1}{5\sqrt{3}}\right)$$

If $\frac{dx}{dt} = 8$ & $\frac{dy}{dt} = -2$

@ $(r, \theta) = (6, \pi/4)$, then

$\frac{dr}{dt}$ & $\frac{d\theta}{dt}$ @ $(r, \theta) = (6, \pi/4)$

can be found by solving

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{\partial x}{\partial r}\dfrac{dr}{dt} + \dfrac{\partial x}{\partial \theta}\dfrac{d\theta}{dt} \\[2ex] \dfrac{dy}{dt} = \dfrac{\partial y}{\partial r}\dfrac{dr}{dt} + \dfrac{\partial y}{\partial \theta}\dfrac{d\theta}{dt} \end{cases}$$

Is $\left[\frac{x+y}{x^2+y^2} \text{ if } (x,y)\neq(0,0) \text{ else } \underline{0}\right]$

continuous at $(0,0)$?

① Try $(x,y)\gtrsim(0,0)$ but $\neq(0,0)$:

$$\frac{.0001+0}{(.0001)^2+0^2} = \frac{10^{-4}}{10^{-8}} = 10^4 = 10{,}000$$

not close to $\underline{0} \Rightarrow$ probably not cts.

$$f(x,y) = \begin{cases} \dfrac{x+y}{x^2+y^2} & : (x,y)\neq(0,0) \\ 0 & : (x,y)=(0,0) \end{cases}$$

Or: ② $$\frac{x+y}{x^2+y^2} = \frac{r\cos\theta + r\sin\theta}{r^2} = \frac{\cos\theta+\sin\theta}{r}$$

$\longrightarrow \pm\infty$ as $r\to 0$ if $\cos\theta+\sin\theta$ fixed $\neq 0$

$$f(x,y) = \begin{cases} \dfrac{3x^3+4y^2+4x^2}{x^2+y^2} & : (x,y) \neq (0,0) \\ 4 & : (x,y) = (0,0) \end{cases}$$

Is f cts. @ $(0,0)$?

$$f(x,y) = \frac{3x^3}{x^2+y^2} + 4\cancel{\left(\frac{x^2+y^2}{x^2+y^2}\right)}$$

Polar approach: $\dfrac{|x|^3}{x^2+y^2} \leq \dfrac{r^3}{r^2} = r \to 0$

as $r \to 0$, so f is cts @ $(0,0)$

(Why? $|x| = r|\cos\theta| \leq r(1) = r$.)

Numerical approach: Plug in $(x,y) \approx (0,0)$ but $\neq (0,0)$. See if $f(x,y) \approx 4$:

$(x,y) = (.01, .01) \Rightarrow f(x,y) = 4.015.$

$(x-1)$^2 + $(y-2)$^2 + (x^2 + y^2 + 3)^2

$\rightarrow f$

$\underbrace{\partial(f, x) \rightarrow fx}_{89}$ OR $\underbrace{\frac{d}{dx} f \rightarrow fx}_{\text{NSPire CAS}}$

$\partial(f, y) \rightarrow fy$ OR $\frac{d}{dy} f \rightarrow fy$

solve(fx = 0 and fy = 0, {x, y})

(Calculator steps for HW 27 #1.)

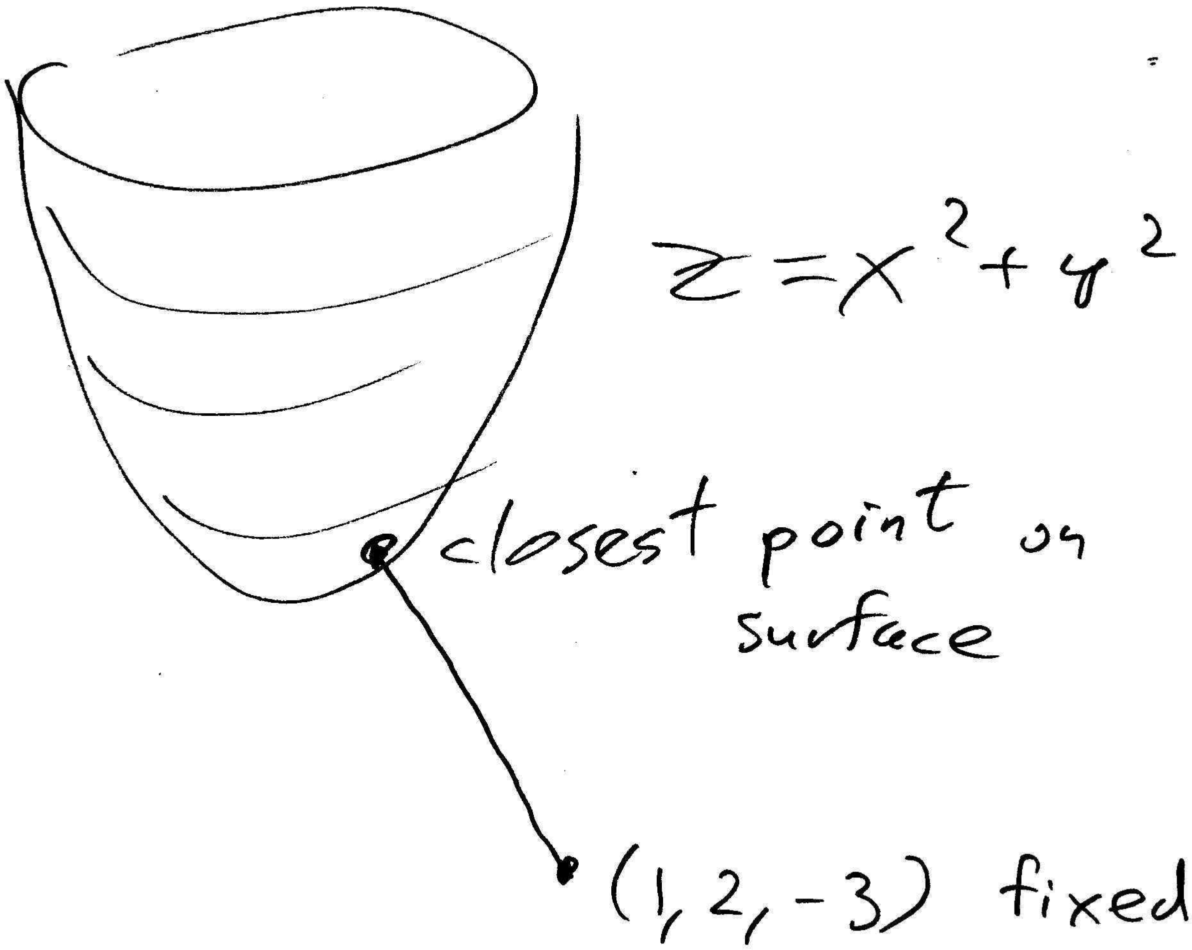
$x = 0.139\ldots$ & $y = 0.278\ldots$

$\Rightarrow$ z = x^2 + y^2 = $0.0966\ldots$

$(x, y, z) = (0.139\ldots, 0.278\ldots, 0.0966\ldots)$

is closest on $z = x^2 + y^2$ surface

to $(1, 2, -3)$.


$z = x^2 + y^2$
closest point on surface
(1, 2, -3) fixed

SP17 Test 3 #1

$$f(x,y) = \frac{x}{\sqrt{y}} = x y^{-1/2}$$

$$(a,b) = (6,4) \qquad f(a,b) = \frac{6}{\sqrt{4}} = 3$$

$f(\underbrace{6.04}_{a+dx},\ \underbrace{3.99}_{b+dy}) \approx ?$ via tangent approx.

$b+dy \to dy = -0.01 (= 3.99 - 4)$

$a+dx \to dx = 0.04\ (= 6.04 - 6)$

$$f(x,y) \approx \underbrace{f(a,b)}_{3} + f_x(a,b)\underbrace{dx}_{.04} + f_y(a,b)\underbrace{dy}_{-.01}$$

$$\frac{\partial}{\partial x}(x y^{-1/2}) = 1 y^{-1/2} = f_x = 4^{-1/2} = \frac{1}{2}$$

@(a,b)

$$\frac{\partial}{\partial y}(x y^{-1/2}) = x\left(-\tfrac{1}{2}\right)y^{-\frac{3}{2}}$$

$$= 6\left(-\tfrac{1}{2}\right)4^{-3/2} = -3 \cdot 4^{-3/2}$$

$$= -3/4^{3/2} = -3/(\sqrt{4})^3$$

$$= -3/2^3 = -3/8$$

$$f(x,y) \approx \boxed{3 + \tfrac{1}{2}(.04) + \left(-\tfrac{3}{8}\right)(-.01)}$$

$$= 3 + .02 + .00375 = \boxed{3.02375}$$

~~3~~ F16 Test 3 #1

$\vec{\nabla}(x^6y^3-2z) \perp$ tangent plane

@ (x,y,z)

for surface

$x^6y^3-2z=\text{constant}$

$$\vec{\nabla}(\ldots)=\left\langle \frac{\partial}{\partial x}(\ldots), \frac{\partial}{\partial y}(\ldots), \frac{\partial}{\partial z}(\ldots)\right\rangle$$

$$=\langle 6x^5y^3, 3x^6y^2, -2\rangle$$

$$=\boxed{\langle 6, 3, -2\rangle} \text{ @ } (1,1,0)$$

($x^6y^3-2z=1$ @ $(1,1,0)$,

but we don't use the "$=1$".)

SP16 Test 3 #1:

Another gradient problem:

$$T = 3x^{-9}y^{4}z^{-2}$$

(ii) At $(1,1,1)$, $\frac{\vec{\nabla} T}{|\vec{\nabla} T|}$ is the direction of fastest increase:

$$\vec{\nabla} T = \langle T_x, T_y, T_z \rangle$$

$$= \langle -27x^{-10}y^{4}z^{-2}, \; 12x^{-9}y^{3}z^{-2}, \; -6x^{-9}y^{4}z^{-3} \rangle$$

$$= \langle -27, 12, -6 \rangle \; @ \; (1,1,1)$$

$$= 3\langle -9, 4, -2 \rangle$$

$$\frac{\vec{\nabla} T}{|\vec{\nabla} T|} = \frac{\langle -9, 4, -2 \rangle}{\sqrt{9^2+4^2+2^2}} \approx \langle -.896, .398, -.199 \rangle$$