

Hidden parabolas.

1. Choose three integer coefficients a, b, c and random noise $e_1, e_2, \dots, e_{11} \in \{-10, -9, -8, \dots, 10\}$.
2. Compute $y_i = ax_i^2 + bx_i + c + e_i$ for each of the integers $x_1 = -5, x_2 = -4, \dots, x_{11} = 5$.
3. Write just your list y_1, y_2, \dots, y_{11} as a column vector \mathbf{y} on a separate sheet of paper.
4. Keep your a, b, c, \dots sheet to yourself but trade your y -list sheet with a partner. (Cyclic trades between more than two people are also fine.)
5. Use least squares to estimate $\hat{a}, \hat{b}, \hat{c}$ of your partner's a, b, c :

(a) Form the matrix $A = \begin{bmatrix} 1 & -5 & (-5)^2 \\ 1 & -4 & (-4)^2 \\ 1 & -3 & (-3)^2 \\ \vdots & \vdots & \vdots \\ 1 & 5 & 5^2 \end{bmatrix}$. (I recommend storing A as a variable or table on your calculator.)

(b) Compute $\begin{bmatrix} \hat{c} \\ \hat{b} \\ \hat{a} \end{bmatrix} = (A^T A)^{-1} A^T \mathbf{y}$. (\hat{c} comes first here because A has the 1s column first. If you want \hat{a} to come first, put the squares column first in A .)