MATH 4335 TEST 1

Name: ____________________________

Date: October 1, 2012.

Exercise	Point Possible	Score
1	48	
2	11	
3	11	
4	30	
Total	100	

1. [48 points] Fill in the following rectangles with "yes" or "no" as appropriate for describing the following six subsets of $\mathbb{R}$.

	$[4,6] \cup [7,8]$	$(1,2) \cup (3,4)$	$\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\} \cup \{1\}$	$(-\infty, 2)$	$[3, \infty)$	$\mathbb{Q}$
bounded?						
open?						
closed?						
compact?						
connected?						
perfect?						
countable?						
dense?						

2. [11 points] Given an example of a bounded set of rationals with an irrational least upper bound.

3. [11 points] Give an example of a bounded sequence of reals that does not converge.

4. [30 points] Let $x_1 = 1$ and $x_{n+1} = \sqrt[3]{x_n^2 + 3}$ for all $n = 1, 2, 3, \dots$

(a) (10 pts) Assuming $0 < x_n < x_{n+1} < 2$ for all n, prove that $x_1, x_2, x_3, \dots$ converges in $\mathbb{R}$.

(b) (20 pts) Prove by induction on n that $0 < x_n < x_{n+1} < 2$ for all n.

(c) (10 pts extra credit) Assuming $\lim_{n\to\infty} x_n = y$, prove that $y = \sqrt[3]{y^2 + 3}$.