MATH 4335 TEST 1

Name: ______________________________

Date: Oct. 2, 2013.

Exercise	Point Possible	Score
1	20	
2	40	
3	40	
Total	100	

1. [20 points] Consider these two sequences:

$$a_1, a_2, a_3, \ldots = \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \ldots$$

$$b_1, b_2, b_3, \ldots = \frac{1}{3}, \frac{2}{3}, \frac{2}{5}, \frac{3}{5}, \frac{3}{7}, \frac{4}{7}, \frac{4}{9}, \frac{5}{9}, \frac{5}{11}, \frac{6}{11}, \ldots$$

(a) (5pts) Which of these sequences are bounded?

(b) (5pts) Which of these sequences are monotone?

(c) (10pts) Which of these sequences converge?

Proofs are optional for this question.

2. [40 points] Recursively define a sequence $\{x_n\}$ by $x_1 = 1$, $x_2 = 6$, and

$$x_{n+2} = (9x_{n+1} - 4x_n)/5.$$

Prove that this sequence is Cauchy.

3. [40 points] Prove that if $L > 0$, $p_n \geq 0$ for all n, and $p_n \to L$, then $\sqrt{p_n} \to \sqrt{L}$.
Hint: $\sqrt{x} - \sqrt{y} = \frac{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}{\sqrt{x}+\sqrt{y}}$.