# MATH 4335-101 FINAL EXAM

Name: ______________________________

Testing conditions:
Materials allowed: calculator, pen, pencil, eraser, etc.
(Actually, for the selected problems, a calculator is not needed.)
Prohibited materials: notes, books, phones, etc.

*Date*: Dec. 8, 2014.

| Exercise | Point Possible | Score |
|---|---|---|
| 1 | 33 | |
| 2 | 33 | |
| 3 | 34 | |
| Total | 100 | |

**1. [33 points]** Let $y_1 = 20$, and $y_{n+1} = (y_n/3) + 5$. Prove that $(y_n)_{n\in\mathbb{N}}$ is decreasing and bounded below.

**2. [33 points]** Let $f$, $g$, and $h$ be partial functions from $\mathbb{R}$ to $\mathbb{R}$. For each of the following statements, if it's false, provide a counterexample.

(1) If $f$ is invertible and bounded, then $f^{-1}$ is bounded.
(2) If $f$ is invertible and decreasing, then $f^{-1}$ is decreasing.
(3) if $f$ is increasing and $g$ and $h$ are decreasing, then $f \circ (g \circ h)$ is increasing.
(4) If $f$ is bounded, then $f \circ g$ is bounded.
(5) If $f$ is bounded, then $g \circ f$ is bounded.
(6) If $\mathrm{dom}(f) = \mathrm{dom}(g) = \mathbb{R}$, then $\mathrm{dom}(f \circ g) = \mathbb{R}$.

**3. [34 points]** Prove that if $f\colon (0,1) \to \mathbb{R}$ is uniformly continuous, then $\lim_{x\to 0^+} f(x)$ exists.