MATH 4335 FINAL

Name: ______________________________

Date: Dec. 15, 2015.

1.

(a) Let $\{a_n\}$ be a sequence, and let $\{a_{n_i}\}$ be any subsequence. Prove that if $\sum_{n=0}^{\infty} a_n$ is absolutely convergent, then $\sum_{i=0}^{\infty} a_{n_i}$ is absolutely convergent

(b) Show by counterexample that the above is false if the word "absolutely" is dropped everywhere.

2.

(a) Give an example of a bounded function $f\colon \mathbb{R} \to \mathbb{R}$.
(b) Give an example of a function $g\colon \mathbb{R} \to \mathbb{R}$ that is locally bounded but not bounded.
(c) Give an example of a function $h\colon \mathbb{R} \to \mathbb{R}$ that is not locally bounded.

3. Suppose that $f'(x) = 3g(x)$ and $g'(x) = 4f(x)$ for all $x \in \mathbb{R}$. Prove that between any two zeroes of f is a zero of g.

4. Suppose that $a < b$, f is continuous on $[a, b]$, $f \geq 0$ on $[a, b]$, and $f(c) > 0$ for at least one point on $[a, b]$.

(a) Show that $\int_a^b f(x)\,dx > 0$.

(b) Give an example of a function g such that g is integrable on $[0, 1]$, $g \geq 0$ on $[0, 1]$, $g(0) > 0$, and $\int_0^1 g(x)\,dx = 0$.

5. Prove that if $n \in \{2, 3, 4, \ldots\}$ and $0 < x$, then

$$\left(1 + \frac{x}{n}\right)^n < \sum_{k=0}^{n} \frac{x^k}{k!}.$$