Topology

Picture:

Invariants with respect to stretching, squeezing, bending. (No puncturing or tearing, glueing)

General Topology = Point-set Topology

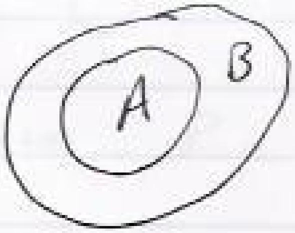
• The ~~language~~ language of sets is today's topic:

①

$a \in A$

"is in" set

②



$A \subset B$

SAME as $A \subseteq B$

"is a subset of"

• everything in A is also in B

③ $A = B$ for sets means $A \subseteq B \subseteq A$.

④ $A \cup B$

$A \cup B = \{ p \mid p \in A \text{ or } p \in B \}$

= everything in A, B, or both

"P or Q" means P, Q or both.

⇒

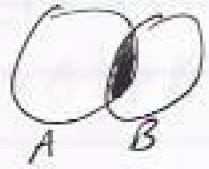
⑤ Set notation:

$\{x \mid P\}$

is set of all x with property P.

⑥ $A \cap B$

"Intersection"



$A \cap B = \{p \mid p \in A \text{ and } p \in B\}$.

⑦ $\emptyset$ = empty set

ex:

$\Rightarrow C \cap D = \emptyset$

"If P, then Q" means P is false, Q is true, or both.

$P \Rightarrow Q$

means "If P, then Q"

$P \Leftrightarrow Q$ means: $P \Rightarrow Q \Rightarrow P$; also expressed as:

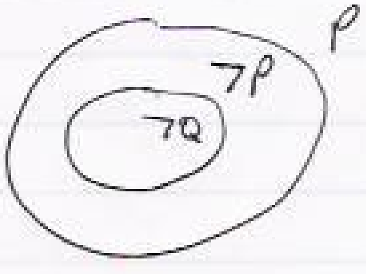
"P if and only if Q"

The contrapositive

of $P \Rightarrow Q$

is $(\neg Q) \Rightarrow (\neg P)$

$\neg$ = "not"



$(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$

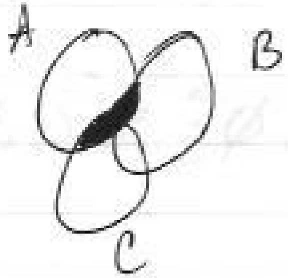
— More set Notation

- $A - B$ same as $A \setminus B$

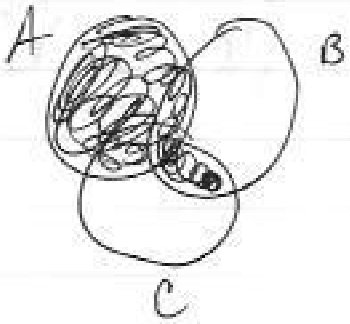
same as $\{p \mid p \in A \text{ and } p \notin B\}$

same as $\{p \in A \mid p \notin B\}$

- Distributive laws
  - $x(y+z) = xy + xz$
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

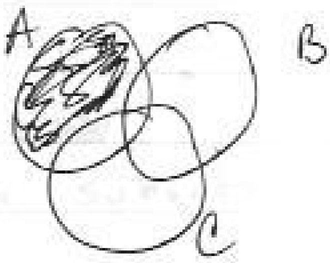


- Also: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

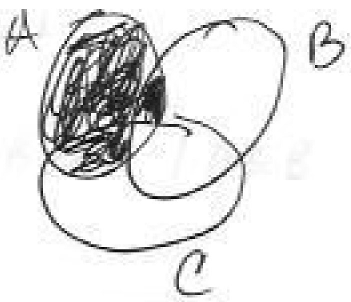


*(in numbers => $x + yz \neq (x+y)(x+z)$)

- $A - (B \cup C) = (A - B) \cap (A - C)$ => DeMorgan's Law



$A-(B\cap C) = (A-B)\cup(A-C)$ => De Morgan's Law



- The power set of A is:

$$\{B \mid B \subset A\}$$

Notation: $P(A)$

$$P(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$$

$$a\in A \Leftrightarrow \{a\}\subseteq A \Leftrightarrow \{a\}\in P(A)$$

• $P(P(\{a,b\})) = \{\emptyset, \{\{a\}\}, \{\{b\}\}, \{\{a,b\}\}, \{\emptyset,\{a\}\},$
$\{\emptyset,\{b\}\}, \{\emptyset,\{a,b\}\},$
$\{\emptyset,\{a\},\{b\}\}, \{\emptyset,\{a\},\{a,b\}\}$
$\{\emptyset,\{b\},\{a,b\}\}, \{\{a\},\{b\},\{a,b\}\}$
$\{\{a\},\{b\}\}, \{\{a\},\{a,b\}\}, \{\{b\},\{a,b\}\}$
$\{\emptyset,\{a\},\{b\},\{a,b\}\}\ \{\emptyset\}\}$

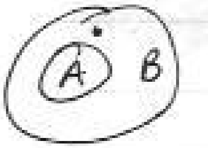
~~Proper~~

- Proper subsets =>

(Cont'd)

$A \subsetneq B$

$A \subsetneq B \iff (A \subseteq B \text{ and } A = B)$



$\{B \mid B \subsetneq A\} = P(A) - \{A\}$

$P(\emptyset) - \{\emptyset\} = \emptyset$ because $P(\emptyset) = \{\emptyset\}$

$\bigcap_{i=1}^{17} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_{17}$

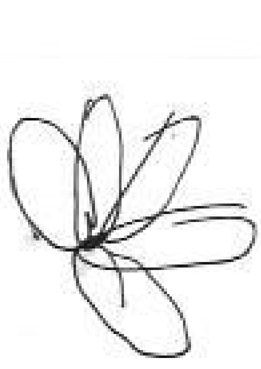
$\bigcup_{i=0}^{5} B_i = B_0 \cup B_1 \cup B_2 \cup \cdots B_5$

$\bigcap_{i=0}^{\infty} C_i = C_0 \cap C_1 \cap C_2 \cap \ldots C_\infty$

Ex: $\bigcap_{A \in \mathcal{A}} A = \{p \mid \text{for all } A \in \mathcal{A},\ p \in A\}$

$\Downarrow$

$\bigcap \mathcal{A}$



$\bigcup \mathcal{A} = \bigcup_{A \in \mathcal{A}} A$

$= \{p \mid \text{for some } A \in \mathcal{A},\ p \in A\}$

$\Rightarrow \bigcup_{A \in \mathcal{A}} B_A = \{p \mid \text{for some } A \in \mathcal{A},\ p \in B_A\}$

Ex: $\bigcup_{A \in P(B)} \{A, B\}$ $\Leftarrow$ random example

Practice: write out $\bigcup_{A \in P(\{0,1\})} \{A, 0\}$

– $A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$